

Scilab Textbook Companion for
An Introduction To Numerical Analysis
by K. E. Atkinson¹

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Book Description

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Scilab numbering policy used in this document and the relation to the above book.

Exa Example (Solved example)

Eqn Equation (Particular equation of the above book)

AP Appendix to Example(Scilab Code that is an Appednix to a particular Example of the above book)

For example, Exa 3.51 means solved example 3.51 of this book. Sec 2.3 means a scilab code whose theory is explained in Section 2.3 of the book.

Contents

List of Scilab Codes	4
1 Error Its sources Propagation and Analysis	8
2 Rootfinding for Nonlinear equations	15
3 Interpolation Theory	25
4 Approximation of functions	31
5 Numerical Integration	41
6 Numerical methods for ordinary differential equations	54
7 Linear Algebra	62
8 Numerical solution of systems of linear equations	66
9 The Matrix Eigenvalue Problem	76

List of Scilab Codes

Exa 1.1	Taylor series	8
Exa 1.3	Vector norms	9
Exa 1.4	Conversion to decimal	9
Exa 1.5	Error and relative error	10
Exa 1.6	Errors	10
Exa 1.7	Taylor series	11
Exa 1.8	Graph of polynomial	11
Exa 1.9	Error and Relative error	12
Exa 1.10	Loss of significance errors	13
Exa 1.11	Loss of significance errors	14
Exa 2.1	Bisection method	15
Exa 2.2	Newton method	16
Exa 2.3	Secant method	17
Exa 2.4	Muller method	18
Exa 2.6	Muller method	19
Exa 2.7	One point iteration method	20
Exa 2.8	One point Iteration method	21
Exa 2.10	Aitken	23
Exa 2.11	Multiple roots	23
Exa 3.1	Lagrange formula	25
Exa 3.2	Lagrange Formula	25
Exa 3.3	Lagrange formula	26
Exa 3.4	Divided differences	26
Exa 3.6	Bessel Function	27
Exa 3.7	Divided differences	28
Exa 3.8	Newton forward difference	28
Exa 4.1	Error of approximating exponent of x	31
Exa 4.2	Minimax Approximation problem	31

Exa 4.3	Least squares approximation problem	33
Exa 4.4	Weight functions	34
Exa 4.5	Formulae	35
Exa 4.6	Formulae for laguerre and legendre polynomials	35
Exa 4.7	Average error in approximation	36
Exa 4.8	Chebyshev expansion coefficients	37
Exa 4.9	Max errors in cubic chebyshev least squares approx	38
Exa 4.10	Near minimax approximation	38
Exa 4.11	Forced oscillation of error	39
Exa 5.1	Integration	41
Exa 5.2	Trapezoidal rule for integration	41
Exa 5.3	Corrected trapezoidal rule	43
Exa 5.4	Simpson s rule for integration	44
Exa 5.5	Trapezoidal and simpson integration	45
Exa 5.6	Newton Cotes formulae	47
Exa 5.7	Gaussian Quadrature	47
Exa 5.8	Gaussian Legendre Quadrature	48
Exa 5.9	Integration	49
Exa 5.10	Simpson Integration error	50
Exa 5.11	Romberg Integration	51
Exa 5.12	Adaptive simpson	51
Exa 5.13	Integration	52
Exa 5.14	Integration	52
Exa 6.1	1st order linear differential equation	54
Exa 6.4	Stability of solution	54
Exa 6.5	Euler method	55
Exa 6.6	Euler	56
Exa 6.7	Asymptotic error analysis	57
Exa 6.9	Midpoint and trapezoidal method	57
Exa 6.10	Euler	58
Exa 6.11	Trapezoidal method	59
Exa 6.16	Adams Moulton method	59
Exa 6.21	Euler method	60
Exa 6.24	Trapezoidal method	60
Exa 6.31	Boundary value problem	61
Exa 7.1	Orthonormal basis	62
Exa 7.2	Canonical forms	62
Exa 7.3	Orthonormal eigen vectors	63

Exa 7.4	Vector and matrix norms	63
Exa 7.5	Frobenious norm	64
Exa 7.6	Norm	64
Exa 7.7	Inverse exists	65
Exa 8.2	LU decomposition	66
Exa 8.4	LU decomposition	67
Exa 8.5	Choleski Decomposition	67
Exa 8.6	LU decomposition	68
Exa 8.7	Error analysis	69
Exa 8.8	Residual correction method	70
Exa 8.9	Residual correction method	72
Exa 8.10	Gauss Jacobi method	72
Exa 8.11	Gauss seidel method	73
Exa 8.13	Conjugate gradient method	74
Exa 9.1	Eigenvalues	76
Exa 9.2	Eigen values and matrix norm	76
Exa 9.3	Bounds for perturbed eigen values	77
Exa 9.4	Eigenvalues of nonsymmetric matrix	79
Exa 9.5	Stability of eigenvalues for nonsymmetric matrices	80
Exa 9.7	Rate of convergence	81
Exa 9.8	Rate of convergence after extrapolation	82
Exa 9.9	Householder matrix	82
Exa 9.11	QR factorisation	83
Exa 9.12	Tridiagonal Matrix	84
Exa 9.13	Planner Rotation Orthogonal Matrix	84
Exa 9.14	Eigen values of a symmetric tridiagonal Matrix	84
Exa 9.15	Sturm Sequence property	85
Exa 9.16	QR Method	85
Exa 9.18	Calculation of Eigen vectors and Inverse iteration	86
Exa 9.19	Inverse Iteration	87
AP 1	Gauss seidel method	88
AP 2	Euler method	88
AP 3	Eigen vectors	89
AP 4	Boundary value problem	90
AP 5	Trapezoidal method	91
AP 6	Legendre Polynomial	92
AP 7	Romberg Integration	93
AP 8	Lagrange	94

AP 9	Muller method	95
AP 10	Secant method	96
AP 11	Newton	96
AP 12	Aitken1	97
AP 13	Bisection method	97

Chapter 1

Error Its sources Propagation and Analysis

Scilab code Exa 1.1 Taylor series

```
1          // PG (6)
2
3 // Taylor series for  $e^{-x^2}$  upto first four
  terms
4
5 deff ('[y]=f(x)', 'y=exp(-x^2)')
6 funcprot(0)
7 deff ('[y]=fp(x)', 'y=-2*x*exp(-x^2)')
8 funcprot(0)
9 deff ('[y]=fpp(x)', 'y=(1-2*x^2)*(-2*exp(-2*x^2))')
10 funcprot(0)
11 deff ('[y]=g(x)', 'y=4*x*exp(-x^2)*(3-2*x^2)')
12 funcprot(0)
13 deff ('[y]=gp(x)', 'y=(32*x^4*exp(-x^2))+(-72*x^2*exp
    (-x^2))+12*exp(-x^2)')
14 funcprot(0)
15 x0=0;
16 x=poly(0, "x");
17 T = f(x0) + (x-x0)*fp(x0)/factorial(1) + (x-x0)^2 *
```

```

    fpp(x0)/factorial(2) + (x-x0)^3 * g(x0)/factorial
    (3) + (x-x0)^4 * gp(x0)/factorial(4)
18
19
20
21 //      Similarly Taylor series for inv(tan(x))

```

Scilab code Exa 1.3 Vector norms

```

1          //      PG (11)
2
3  A = [1 -1;3 2]
4  x = [1;2]
5  y = A*x
6  norm(A, 'inf')
7  norm(x, 'inf')
8  norm(y, 'inf')
9
10 x = [1;1]
11 y = A*x
12 norm(y, 'inf')
13 norm(A, 'inf')*norm(x, 'inf')
14
15 //      norm(y, 'inf') = norm(A, 'inf') * norm(x, 'inf')

```

Scilab code Exa 1.4 Conversion to decimal

```

1          //      PG (12)
2
3  //      11011.01 is a binary number. Its decimal
    equivalent is:
4

```

```

5 1*2^4 + 1*2^3 + 0*2^2 + 1*2^1 + 1*2^0 + 0*2^(-1) +
  1*2^(-2)
6
7 //      56C.F is a hexadecimal number. Its decimal
  equivalent is :
8
9 5*16^2 + 6*16^1 + 12*16^0 + 15*16^(-1)

```

Scilab code Exa 1.5 Error and relative error

```

1          //      PG (17)
2
3 xT = exp(1)
4 xA = 19/7
5
6 //      Error(xA)
7
8 xT - xA
9
10 //      Relative error , Rel(xA)
11
12 (xT-xA)/xT

```

Scilab code Exa 1.6 Errors

```

1          //      PG (18)
2
3 xT = 1/3
4 xA = 0.333
5 abs(xT-xA)      //      Error
6
7 //-----
8

```

```

9  xT = 23.496
10 xA = 23.494
11 abs(xT-xA)    //      Error
12
13 //-----
14
15 xT = 0.02138
16 xA = 0.02144
17 abs(xT-xA)    //      Error

```

Scilab code Exa 1.7 Taylor series

```

1          //      PG (20)
2
3 //      Taylor series for the first two terms
4
5 deff(' [y]=f(x) ', 'y=sqrt(1+x) ')
6 funcprot(0)
7 deff(' [y]=fp(x) ', 'y=0.5*(1+x)^(-1/2) ')
8 funcprot(0)
9 x0=0;
10 x=poly(0, "x");
11 T = f(x0) + (x-x0)*fp(x0)/factorial(1)

```

Scilab code Exa 1.8 Graph of polynomial

```

1          //      PG (21)
2
3 deff(' [y]=f(x) ', 'y = x^3-3*x^2+3*x-1 ')
4 xset('window', 0);
5 x=-0:.01:2;

```

//

defining the range of x.

```

6 y=feval(x,f);
7
8 a=gca();
9
10 a.y_location = "origin";
11
12 a.x_location = "origin";
13 plot(x,y)

    // instruction to plot the graph
14
15 title(' y = x^3-3*x^2+3*x-1')

```

Scilab code Exa 1.9 Error and Relative error

```

1          // PG (24)
2
3 xT = %pi
4 xA = 3.1416
5 yT = 22/7
6 yA = 3.1429
7 xT - xA          // Error
8 (xT - xA)/xT    // Relative Error
9 yT - yA          // Error
10 (yT - yA)/yT   // Relative Error
11
12 (xT - yT) - (xA - yA)
13 ((xT - yT) - (xA - yA))/(xT - yT)
14
15 // Although the error in xA - yA is quite small,
16 // the relative error in xA - yA is much larger
    than that in xA or yA alone.

```

Scilab code Exa 1.10 Loss of significance errors

```
1          // PG (25)
2
3 // Consider solving  $ax^2 + b*x + c =$ 
4
5
6 // Consider a polynomial  $y = x^2 - 26*x + 1 = 0$ 
7
8 x = poly(0,"x");
9 y = x^2 - 26*x + 1
10 p = roots(y)
11 ra1 = p(2,1)
12 ra2 = p(1,1)
13
14 // Using the standard quadratic formula for
    finding roots ,
15
16 rt1 = (-(-26)+sqrt((-26)^2 - 4*1*1))/(2*1)
17 rt2 = (-(-26)-sqrt((-26)^2 - 4*1*1))/(2*1)
18
19 // Relative error
20
21 rel1 = (ra1-rt1)/ra1
22 rel2 = (ra2-rt2)/ra2
23
24 // The significant errors have been lost in the
    subtraction  $ra2 = xa - ya$ .
25 // The accuracy in ra2 is much less.
26 // To calculate ra2 accurately , we use:
27
28 rt2 = ((13-sqrt(168))*(13+sqrt(168)))/(1*(13+sqrt
    (168)))
29 // Now, rt2 is nearly equal to ra2. So, by exact
    calculations , we will now get a much better rel2.
```

Scilab code Exa 1.11 Loss of significance errors

```
1          // PG (26)
2
3  x = poly(0,"x");
4  x = 0;
5  deff(' [y]=f(t)', 'y=exp(x*t)')
6  integrate('exp(x*t)', 't', 0, 1)
7
8  // So, for x = 0, f(0) = 1
9  // f(x) is continuous at x = 0.
10
11 // To see that there is a loss of significance
12 // problem when x is small,
13 // we evaluate f(x) at 1.4*10(-9)
14 x = 1.4*10(-9)
15 integrate('exp(x*t)', 't', 0, 1)
16 // When we use a ten-digit hand calculator, the
17 // result is 1.000000001
18 // To avoid the loss of significance error, we
19 // may use a quadratic Taylor approximation to exp(x
20 // ) and then simplify f(x).
```

Chapter 2

Rootfinding for Nonlinear equations

check Appendix [AP 13](#) for dependency:

```
bisection1.sce
```

Scilab code Exa 2.1 Bisection method

```
1 // EXAMPLE (PG 57)
2 // To find largest root, alpha, of  $x^6 - x - 1 = 0$ 
3 // using bisection method
4 // The graph of this function can also be
   observed here.
5
6 def ('[y]=f(x)', 'y=x^6-x-1')
7 // It is straightforward to
   show that  $1 < \alpha < 2$ , and
8 //we will use this as our
   initial interval [a,b]
9
10
11 xset('window',0);
```



```

12 x=-5:.01:5;
//
    defining the range of x.
13 y=feval(x,f);
14
15 a=gca();
16
17 a.y_location = "origin";
18
19 a.x_location = "origin";
20 plot(x,y)

    // instruction to plot the graph
21
22 title(' y = x^6-x-1')
23
24 // execution of the user defined function so as to
    use it in program to find the approximate
    solution.
25
26 // we call a user-defined function 'bisection' so as
    to find the approximate
27 // root of the equation with a defined permissible
    error.
28
29 bisection(1,2,f)

```

check Appendix [AP 11](#) for dependency:

newton.sce

Scilab code Exa 2.2 Newton method

```

1 // EXAMPLE (PG 60)
2 // To find largest root, alpha, of  $f(x) = x^6$ 
    -  $x - 1 = 0$ 

```

```

3      //      using newton's method
4
5
6  def f(' [y]=f(x) ', 'y=x^6-x-1')
7  def fp(' [y]=fp(x) ', 'y=6*x^5-1') //
      Derivative of f(x)
8  x=(1+2)/2 //      Initial
      appoximation
9
10 //we call a user-defined function 'newton' so as to
      find the approximate
11 // root of the equation with a defined permissible
      error.
12
13
14 newton(x,f,fp)

```

check Appendix [AP 10](#) for dependency:

secant.sce

Scilab code Exa 2.3 Secant method

```

1      //      EXAMPLE ( PG 66)
2      //      To find largest root, alpha, of  $f(x) = x^6$ 
      -  $x - 1 = 0$ 
3      //      using secant method
4
5  def f(' [y]=f(x) ', 'y=x^6-x-1')
6  a=1
7  b=2 //      Initial approximations
8
9
10 // we call a user-defined function 'secant' so as to
      find the approximate

```

```

11 // root of the equation with a defined permissible
    error.
12
13 secant(a,b,f)

```

check Appendix [AP 9](#) for dependency:

muller.sce

Scilab code Exa 2.4 Muller method

```

1 // EXAMPLE1 (PG 76)
2 // f(x) = x^20 - 1
3 // solving using Muller's method
4
5
6 xset('window',1);
7 x=-2:.01:4; //
    defining the range of x.
8 deff(' [y]=f(x) ', 'y=x^20-1 '); //
    defining the function.
9 y=feval(x,f);
10
11 a=gca();
12
13 a.y_location = "origin";
14
15 a.x_location = "origin";
16 plot(x,y) //
    instruction to plot the graph
17 title(' y = x^20-1')
18
19 // from the above plot we can infere that the
    function has roots between
20 // the intervals (0,1),(2,3).
21

```

```

22         //solution by muller method to 3 iterations
23         .
24 muller(0, .5, 1, f)

```

check Appendix [AP 9](#) for dependency:

muller.sce

Scilab code Exa 2.6 Muller method

```

1     //     EXAMPLE3 (PG 76)
2     //     f(x) = x^6- 12 * x^5 + 63 * x^4 - 216* x^3
           + 567 * x^2 - 972 * x + 729
3     //     or f(x) = (x^2+9)*(x-3)^4
4     //     solving using Muller 's method
5
6     deff ('[y]=f(x)', 'y=(x^2+9)*(x-3)^4')
7
8     xset('window', 2);
9     x=-10:.1:10;
                                     //
           defining the range of x.
10    y=feval(x, f);
11
12    a=gca();
13
14    a.y_location = "origin";
15
16    a.x_location = "origin";
17    plot(x, y)
                                     // instruction to plot the graph
18
19    title(' y = (x^2+9)*(x-3)^4')
20

```

```
21
22 muller(0,.5,1,f)
```

Scilab code Exa 2.7 One point iteration method

```
1 // EXAMPLE (PG 77)
2 // x^2-a = 0
3
4 // The graph for x^2-3 can also be observed
   here.
5
6 deff(' [y]=f(x) ', 'y=x*x-3')
7 funcprot(0)
8 xset('window',3);
9 x=-2:.01:10;
                                     //
   defining the range of x.
10 y=feval(x,f);
11
12 a=gca();
13
14 a.y_location = "origin";
15
16 a.x_location = "origin";
17 plot(x,y)
                                     // instruction to plot the graph
18
19 title(' y = x^2-3')
20 // CASE 1
21
22 //We have f(x) = x^2-a.
23 //So, we assume g(x) = x^2+x-a and the value of a =
   3
24
```

```

25 deff(' [y]=g(x) ', 'y=x^2+x-3')
26 funcprot(0)
27 x=2
28 for n=0:1:3
29     g(x);
30     x=g(x)
31 end
32
33 //          CASE 2
34
35 //We have  $f(x) = x^2 - a$ .
36 //So, we assume  $g(x) = a/x$  and the value of  $a = 3$ 
37
38 deff(' [y]=g(x) ', 'y=3/x')
39 funcprot(0)
40 x=2
41 for n=0:1:3
42     g(x);
43     x=g(x)
44 end
45
46 //          CASE 3
47
48 //We have  $f(x) = x^2 - a$ .
49 //So, we assume  $g(x) = 0.5*(x+(a/x))$  and the value
    of  $a = 3$ 
50
51 deff(' [y]=g(x) ', 'y=0.5*(x+(3/x))')
52 funcprot(0)
53 x=2
54 for n=0:1:3
55     g(x);
56     x=g(x)
57 end

```

Scilab code Exa 2.8 One point Iteration method

```
1      //      EXAMPLE (PG 81)
2
3      //Assume alpha is a solution of  $x = g(x)$ 
4
5      alpha=sqrt(3);
6
7      //      case 1
8
9
10     deff(' [y]=g(x) ', 'y=x^2+x-3')
11     deff(' [z]=gp(x) ', 'z=2*x+1')           //      Derivative
12     of g(x)
13     gp(alpha)
14
15     //      case 2
16
17     deff(' [y]=g(x) ', 'y=3/x')
18     funcprot(0)
19     deff(' [z]=gp(x) ', 'z=3/x')           //      Derivative of
20     g(x)
21     gp(alpha)
22
23     //      case 3
24
25     deff(' [y]=g(x) ', 'y=0.5*(x+(3/x))')
26     funcprot(0)
27     deff(' [z]=gp(x) ', 'z=0.5*(1-(3/(x^2)))')           //
28     Derivative of g(x)
29     gp(alpha)
```

check Appendix [AP 12](#) for dependency:

aitken1.sce

Scilab code Exa 2.10 Aitken

```
1 // EXAMPLE (PG 85)
2
3 // x(n+1) = 6.28 + sin(x(n))
4 // True root is alpha = 6.01550307297
5
6 def f ('[y]=f(x)', 'f(x)=6.28+sin(x(n))')
7 // k=6.01550307297
8
9 //x=6.01550307297
10
11 def g ('[y]=g(x)', 'y=cos(x)')
12
13
14 // we call a user-defined function 'aitken' so as to
15 // find the approximate
16 // root of the equation with a defined permissible
17 // error.
18 aitken(0.2,0.5,1,g)
```

Scilab code Exa 2.11 Multiple roots

```
1 // EXAMPLE (PG 87)
2
3 // f(x) = (x-1.1)^3 * (x-2.1)
4
5 c = [2.7951 -8.954 10.56 -5.4 1]
6 p4=poly(c, 'x', 'coeff')
7 roots(p4)
8 def f ('[y]=f(x)', 'y=(x-1.1)^3*(x-2.1)')
9 xset('window',0);
```



```
10 x=0:.01:3;                                     //
    defining the range of x.
11 y=feval(x,f);
12
13 a=gca();
14
15 a.y_location = "origin";
16
17 a.x_location = "origin";
18 plot(x,y)

    // instruction to plot the graph
19
20 title(' y = (x-1.1)^3*(x-2.1)')
```

Chapter 3

Interpolation Theory

check Appendix [AP 8](#) for dependency:

```
lagrange.sce
```

Scilab code Exa 3.1 Lagrange formula

```
1          // PG (134)
2
3 X = [0, -1, 1]
4 Y = [1, 2, 3]
5 lagrange(X,Y)
```

check Appendix [AP 8](#) for dependency:

```
lagrange.sce
```

Scilab code Exa 3.2 Lagrange Formula

```
1          // PG (136)
2
3
```

```

4
5 X=[0]
6 Y=[1]
7 def f(' [y]=f(x) ', 'y=log10(x) ')
8 p=lagrange(X,Y)

```

check Appendix [AP 8](#) for dependency:

lagrange.sce

Scilab code Exa 3.3 Lagrange formula

```

1          // PG (137)
2
3 X=[0, -1]
4 Y=[1, 2]
5 def f(' [y]=f(x) ', 'y=log(x) ')
6 def fp(' [y]=fp(x) ', 'y=1/x ')
7 def fpp(' [y]=fpp(x) ', 'y=-1/(x)^2 ')
8 p = lagrange(X,Y)
9 // E = f(x)-p
10 e = 0.00005 // for a four-place logarithmic
    table

```

Scilab code Exa 3.4 Divided differences

```

1          // PG (140)
2
3 X = [2.0, 2.1, 2.2, 2.3, 2.4]
4 X1 = X(1,1)
5 X2 = X(1,2)
6 X3 = X(1,3)
7 X4 = X(1,4)

```

```

8 X5 = X(1,5)
9 def f(' [y]=f(x) ', 'y=sqrt(x) ')
10 Y = [f(X1) f(X2) f(X3) f(X4) f(X5)]
11 Y1 = Y(1,1)
12 Y2 = Y(1,2)
13 Y3 = Y(1,3)
14 Y4 = Y(1,4)
15 Y5 = Y(1,5)
16
17 //      Difference
18
19 //      f [X1,X2]
20 (f(X2) - f(X1))*10
21 //      f [X2,X3]
22 (f(X3) - f(X2))*10
23 //      f [X3,X4]
24 (f(X4) - f(X3))*10
25 //      f [X4,X5]
26 (f(X5) - f(X4))*10
27
28 //      D^2 * f [Xi]
29
30 ((f(X3)-f(X2)) - (f(X2)-f(X1))) * 50
31 ((f(X4)-f(X3)) - (f(X3)-f(X2))) * 50
32 ((f(X5)-f(X4)) - (f(X4)-f(X3))) * 50

```

Scilab code Exa 3.6 Bessel Function

```

1          //      PG (142)
2
3 //      Values of Bessel Function Jo(x)
4
5 //      x              Jo(x)
6
7 //      2.0          0.2238907791

```

```

8 //          2.1          0.1666069803
9 //          2.2          0.1103622669
10 //         2.3          0.0555397844
11 //         2.4          0.0025076832
12 //         2.5         -0.0483837764
13 //         2.6         -0.0968049544
14 //         2.7         -0.1424493700
15 //         2.8         -0.1850360334
16 //         2.9         -0.2243115458
17
18 //          Calculate the value of x for which Jo(x) = 0.1

```

Scilab code Exa 3.7 Divided differences

```

1 //          PG (144)
2
3 deff ( '[y]=f(x)', 'y=sqrt(x)')
4 funcprot(0)
5 deff ( '[y]=fp(x)', 'y=0.5/sqrt(x)')
6 funcprot(0)
7 deff ( '[y]=fpp(x)', 'y=-0.25*x^(-3/2)')
8 funcprot(0)
9 deff ( '[y]=fppp(x)', 'y=3*x^(-2.5)/8')
10 deff ( '[y]=fpppp(x)', 'y=-15*x^(-7/2)/16')
11
12 //          f[2.0,2.1,.....2.4] = -0.002084
13
14 fpppp(2.3103)/factorial(4)

```

Scilab code Exa 3.8 Newton forward difference

```

1 //          PG (150)
2

```

```

3 X = [2.0,2.1,2.2,2.3,2.4]
4 X1 = X(1,1)
5 X2 = X(1,2)
6 X3 = X(1,3)
7 X4 = X(1,4)
8 X5 = X(1,5)
9 def f(' [y]=f(x) ', 'y=sqrt(x) ')
10 Y = [f(X1) f(X2) f(X3) f(X4) f(X5)]
11 Y1 = Y(1,1)
12 Y2 = Y(1,2)
13 Y3 = Y(1,3)
14 Y4 = Y(1,4)
15 Y5 = Y(1,5)
16
17 //      Difference
18
19 //      f [X1,X2]
20 (f(X2) - f(X1))
21 //      f [X2,X3]
22 (f(X3) - f(X2))
23 //      f [X3,X4]
24 (f(X4) - f(X3))
25 //      f [X4,X5]
26 (f(X5) - f(X4))
27
28 //      D^2 * f [Xi]
29
30 ((f(X3)-f(X2)) - (f(X2)-f(X1)))
31 ((f(X4)-f(X3)) - (f(X3)-f(X2)))
32 ((f(X5)-f(X4)) - (f(X4)-f(X3)))
33
34 //      D^3 * f [Xi]
35
36 ((f(X4)-f(X3)) - (f(X3)-f(X2))) - ((f(X3)-f(X2)) - (
    f(X2)-f(X1)))
37 ((f(X5)-f(X4)) - (f(X4)-f(X3))) - ((f(X4)-f(X3)) - (
    f(X3)-f(X2)))
38

```

```

39 //      D^4 * f[Xi]
40
41 (((f(X5)-f(X4)) - (f(X4)-f(X3))) - ((f(X4)-f(X3)) -
      (f(X3)-f(X2)))) - (((f(X4)-f(X3)) - (f(X3)-f(X2))
      ) - ((f(X3)-f(X2)) - (f(X2)-f(X1))))
42
43 mu = 1.5;
44 x = 2.15;
45
46 p1 = f(X1) + mu * (f(X2) - f(X1))
47 p2 = p1 + mu*(mu-1)*((f(X3)-f(X2)) - (f(X2)-f(X1)))
      /2
48
49 //      Similarly , p3 = 1.466288
50 //      p4 = 1.466288

```

Chapter 4

Approximation of functions

Scilab code Exa 4.1 Error of approximating exponent of x

```
1          // PG (199)
2
3 x = poly(0,"x");
4 p3 = 1 + x + (1/2)*x^(2) + (1/6)*x^3
5 deff(' [y]=f(x) ', 'y=exp(x) ')
6 funcprot(0)
7 x = -1:0.01:1;
8 f(x) - p3
```

Scilab code Exa 4.2 Minimax Approximation problem

```
1          // PG (200)
2
3 deff(' [y]=f(x) ', 'y=exp(x) ')
4
5 xset('window',0);
6 x=-1:.01:1;          // defining the range of
   x.
```



```

7  y=feval(x,f);
8
9  a=gca();
10
11 a.y_location = "origin";
12
13 a.x_location = "origin";
14 plot(x,y)           // instruction to plot the
    graph
15
16
17
18 //     possible approximation
19 //     y = q1(x)
20
21 //     Let e(x) = exp(x) - [a0+a1*x]
22 //     q1(x) & exp(x) must be equal at two points in
    [-1,1], say at x1 & x2
23 //     sigma1 = max(abs(e(x)))
24 //     e(x1) = e(x2) = 0.
25 //     By another argument based on shifting the
    graph of y = q1(x),
26 //     we conclude that the maximum error sigma1 is
    attained at exactly 3 points.
27 //     e(-1) = sigma1
28 //     e(1) = sigma1
29 //     e(x3) = -sigma1
30 //     x1 < x3 < x2
31 //     Since e(x) has a relative minimum at x3, we
    have e'(x) = 0
32 //     Combining these 4 equations, we have..
33 //     exp(-1) - [a0-a1] = sigma1 -----(
    i)
34 //     exp(1) - [a0+a1] = p1 -----(
    ii)
35 //     exp(x3) - [a0+a1*x3] = -sigma1 -----(
    iii)

```

```

36 //      exp(x3) - a1 = 0 -----(
      iv)
37
38 //      These have the solution
39
40 a1 = (exp(1) - exp(-1))/2
41 x3 = log(a1)
42 sigma1 = 0.5*exp(-1) + x3*(exp(1) - exp(-1))/4
43 a0 = sigma1 + (1-x3)*a1
44
45 x = poly(0,"x");
46 //      Thus,
47 q1 = a0 + a1*x
48
49 deff(' [y1]=f(x) ', 'y1=1.2643+1.1752*x ')
50
51 xset('window',0);
52 x=-1:.01:1;          // defining the range of
      x.
53 y=feval(x,f);
54
55 a=gca();
56
57 a.y_location = "origin";
58
59 a.x_location = "origin";
60 plot(x,y)          // instruction to plot the
      graph

```

Scilab code Exa 4.3 Least squares approximation problem

```

1          //      PG (205)
2
3 deff(' [y]=f(x) ', 'y=exp(x) ')
4

```

```

5 x=-1:.01:1;           // defining the range of
  x
6
7 //   Let r1(x) = b0 + b1(x)
8 //   Minimize
9 //       ||f-r1||^2 = integrate('(exp(x)-b0-b1*x)
  ^2','x',-1,1) = F(b0,b1)
10 //   F = integrate('exp(2*x) + b0^2 + (b1^2)*(x^2)
  - 2*b0*x*exp(x) + 2*b0*b1*x','x',b0,b1)
11 //   To find a minimum, we set
12
13 //       df/db0 = 0
14 //       df/db1 = 0-----necessary conditions
  at a minimal point
15 //   On solving, we get the values of b0 & b1
16
17 b0 = 0.5*integrate('exp(x)','x',-1,1)
18 b1 = 1.5*integrate('x*exp(x)','x',-1,1)
19 r1 = b0+b1*x;
20 norm(exp(x)-r1,'inf') //   least squares
  approximation
21
22 r3 = 0.996294 + 0.997955*x + 0.536722*x^2 +
  0.176139*x^3
23 norm(exp(x)-r3,'inf') //   cubic least squares
  approximation

```

Scilab code Exa 4.4 Weight functions

```

1           //   PG (206)
2
3 //   The following are the weight functions of most
  interest in the
4 //   developments of this text:
5

```

```

6 //      w(x)=1          a < = x < = b
7
8 //      w(x)=1/sqrt(1-x^2)  -1 < = x < = 1
9
10 //      w(x)=exp(-x)      0 < = x < infinity
11
12 //      w(x)=exp(-x^2)    -infinity < x <
    infinity

```

Scilab code Exa 4.5 Formulae

```

1 //      PG (215)
2
3 //      for laguerre polynomials ,
4 //      L(n+1)=1*[2*n+1-x]*L(n)/(n+1) - n*L(n-1)/(n+1)
5
6 //      for Legendre polynomials ,
7
8 //      P(n+1)= (2*n +1)*x*P(n)/(n+1) - n*P(n-1)/(n+1)

```

Scilab code Exa 4.6 Formulae for laguerre and legendre polynomials

```

1 //      PG (215)
2
3 //      for laguerre polynomials ,
4 //      L(n+1)=1*[2*n+1-x]*L(n)/(n+1) - n*L(n-1)/(n+1)
5
6 //      for Legendre polynomials ,
7
8 //      P(n+1)= (2*n +1)*x*P(n)/(n+1) - n*P(n-1)/(n+1)

```

Scilab code Exa 4.7 Average error in approximation

```

1           // PG (219)
2
3  def f ('[y]=f(x)', 'y=exp(x)')
4
5  x=-1:.01:1;           // defining the range of
   x
6
7  // Let r1(x) = b0 + b1(x)
8  // Minimize
9  // ||f-r1||^2 = integrate('(exp(x)-b0-b1*x)
   ^2','x',-1,1) = F(b0,b1)
10 // F = integrate('exp(2*x) + b0^2 + (b1^2)*(x^2)
   - 2*b0*x*exp(x) + 2*b0*b1*x','x',b0,b1)
11 // To find a minimum, we set
12
13 // df/db0 = 0
14 // df/db1 = 0-----necessary conditions
   at a minimal point
15 // On solving, we get the values of b0 & b1
16
17 b0 = 0.5*integrate('exp(x)','x',-1,1);
18 b1 = 1.5*integrate('x*exp(x)','x',-1,1);
19 r1 = b0+b1*x;
20 norm(exp(x)-r1,'inf'); // least squares
   approximation
21
22 r3 = 0.996294 + 0.997955*x + 0.536722*x^2 +
   0.176139*x^3;
23 norm(exp(x)-r3,'inf'); // cubic least squares
   approximation
24
25 // average error E
26
27 E = norm(exp(x)-r3,2)/sqrt(2)

```

Scilab code Exa 4.8 Chebyshev expansion coefficients

```
1          // PG (220)
2
3  deff ('[y]=f(x)', 'y=exp(x)')
4
5  // Chebyshev expansion coefficients for exp(x)
6  // j = 0
7  C0=2*(integrate('exp(cos(x))', 'x', 0, 3.14))/(3.14)
8
9  // j = 1
10 C1=2*(integrate('exp(cos(x))*cos(x)', 'x', 0, 3.14))
    /(3.14)
11
12 // j = 2
13 C2=2*(integrate('exp(cos(x))*cos(2*x)', 'x', 0, 3.14))
    /(3.14)
14
15 // j = 3
16 C3=2*(integrate('exp(cos(x))*cos(3*x)', 'x', 0, 3.14))
    /(3.14)
17
18 // j = 4
19 C4=2*(integrate('exp(cos(x))*cos(4*x)', 'x', 0, 3.14))
    /(3.14)
20
21 // j = 5
22 C5=2*(integrate('exp(cos(x))*cos(5*x)', 'x', 0, 3.14))
    /(3.14)
23
24 // we obtain
25 c1=1.266+1.130*x;
26 c3=0.994571+0.997308*x+0.542991*x^2+0.177347*x^3;
27 norm(exp(x)-c1, 'inf')
```

```
28 norm(exp(x)-c3, 'inf')
```

Scilab code Exa 4.9 Max errors in cubic chebyshev least squares approx

```
1          // PG (223)
2
3 deff(' [y]=f(x)', 'y=exp(x)')
4
5 x=[-1.0 -0.6919 0.0310 0.7229 1.0];
6       // defining x
7 r3 = 0.996294 + 0.997955*x + 0.536722*x^2 +
8       0.176139*x^3;
9 norm(exp(x)-r3, 'inf'); // cubic least squares
10 approximation
11 deff(' [y]=g(x)', 'y=0.994571+0.997308*x+0.542991*x
12     ^2+0.177347*x^3')
13 // c3=g(x);
14 x1=x(1,1);
15 (exp(x1)-g(x1))
16 x2=x(1,2);
17 (exp(x2)-g(x2))
18 x3=x(1,3);
19 (exp(x3)-g(x3))
20 x4=x(1,4);
21 (exp(x4)-g(x4))
22 x5=x(1,5);
23 (exp(x5)-g(x5))
```

Scilab code Exa 4.10 Near minimax approximation

```
1          // PG (227)
2
```

```

3 def f(' [y]=f(x) ', 'y=exp(x) ')
4 c3=0.994571+0.997308*x+0.542991*x^2+0.177347*x^3;
5 norm(exp(x)-c3, 'inf')
6
7 //      as obtained in the example 6, c4 = 0.00547, T4
      (x) = (-1)
8 //      c4*T4(x) = 0.00547 * (-1)
9 //      norm(exp(x)-q3, 'inf') = 0.00553

```

Scilab code Exa 4.11 Forced oscillation of error

```

1          //      PG (234)
2
3 def f(' [y]=f(x) ', 'y=exp(x) ')
4 x = -1:0.01:1;
5 //      For
6 n = 1;
7 x = [-1 0 1];
8 E1 = 0.272;
9 F1 = 1.2715 + 1.1752*x;
10
11 //      Relative errors
12
13 x = -1.0;
14 exp(x) - F1;
15 r1 = ans(1,1)
16 x = 0.1614;
17 exp(x) - F1;
18 r2 = ans(1,2)
19 x = 1.0;
20 exp(x) - F1;
21 r3 = ans(1,3)
22
23 F3 = 0.994526 + 0.995682*x + 0.543981*x*x +
      0.179519*x*x*x;

```



```
24 x = [-1.0 -0.6832 0.0493 0.7324 1.0]
25 exp(x) - F3 // relative errors
```

Chapter 5

Numerical Integration

Scilab code Exa 5.1 Integration

```
1          // PG (250)
2
3  defff(' [y]=f(x) ', 'y=(exp(x)-1)/x ')
4  x0=0;
5  x1=1;
6  integrate('(exp(x)-1)/x ', 'x ', x0, x1)
```

Scilab code Exa 5.2 Trapezoidal rule for integration

```
1          // PG (254)
2
3  defff(' [y]=f(x) ', 'y=exp(x)*cos(x) ')
4  defff(' [y]=fp(x) ', 'y=exp(x)*(cos(x)-sin(x)) ')
5  defff(' [y]=fpp(x) ', 'y=-2*exp(x)*sin(x) ')
6  x0=0;
7  x1=%pi;
8
9
```

```

10 //      True value
11 integrate('exp(x)*cos(x)', 'x', x0, x1)
12
13 //      Using Trapezoidal rule
14
15 n=2;
16 h=(x1-x0)/n;
17 I1 = (x1-x0) * (f(x0)+f(x1)) /4
18 E1 = -h^2 * (fp(x1)-fp(x0)) /12
19
20 n=4;
21 h=(x1-x0)/n;
22 I2 = (x1-x0) * (f(x0)+f(x1)) /4
23 E2 = -h^2 * (fp(x1)-fp(x0)) /12
24
25 n=8;
26 h=(x1-x0)/n;
27 I3 = (x1-x0) * (f(x0)+f(x1)) /4
28 E3 = -h^2 * (fp(x1)-fp(x0)) /12
29
30 n=16;
31 h=(x1-x0)/n;
32 I4 = (x1-x0) * (f(x0)+f(x1)) /4
33 E4 = -h^2 * (fp(x1)-fp(x0)) /12
34
35 n=32;
36 h=(x1-x0)/n;
37 I5 = (x1-x0) * (f(x0)+f(x1)) /4
38 E5 = -h^2 * (fp(x1)-fp(x0)) /12
39
40 n=64;
41 h=(x1-x0)/n;
42 I6 = (x1-x0) * (f(x0)+f(x1)) /4
43 E6 = -h^2 * (fp(x1)-fp(x0)) /12
44
45 n=128;
46 h=(x1-x0)/n;
47 I7 = (x1-x0) * (f(x0)+f(x1)) /4

```

48 $E7 = -h^2 * (fp(x1)-fp(x0)) /12$

Scilab code Exa 5.3 Corrected trapezoidal rule

```
1          //    PG (255)
2
3  deff(' [y]=f(x) ', 'y=exp(x)*cos(x) ')
4  deff(' [y]=fp(x) ', 'y=exp(x)*(cos(x)-sin(x)) ')
5  deff(' [y]=fpp(x) ', 'y=-2*exp(x)*sin(x) ')
6  x0=0;
7  x1=%pi;
8
9
10 //    True value
11 integrate('exp(x)*cos(x) ', 'x', x0, x1)
12
13 //    Using Corrected Trapezoidal rule
14
15 n=2;
16 h=(x1-x0)/n;
17 I1 = ((x1-x0)/2) * (f(x0)+f(x1)) /2
18 E1 = -h^2 * (fp(x1)-fp(x0)) /12
19 C1 = I1 + E1
20
21 n=4;
22 h=(x1-x0)/n;
23 I2 = ((x1-x0)/2) * (f(x0)+f(x1)) /2
24 E2 = -h^2 * (fp(x1)-fp(x0)) /12
25 C2 = I2 + E2
26
27 n=8;
28 h=(x1-x0)/n;
29 I3 = ((x1-x0)/2) * (f(x0)+f(x1)) /2
30 E3 = -h^2 * (fp(x1)-fp(x0)) /12
31 C3 = I3 + E3
```

```

32
33 n=16;
34 h=(x1-x0)/n;
35 I4 = ((x1-x0)/2) * (f(x0)+f(x1)) /2
36 E4 = -h^2 * (fp(x1)-fp(x0)) /12
37 C4 = I4 + E4
38
39 n=32;
40 h=(x1-x0)/n;
41 I5 = ((x1-x0)/2) * (f(x0)+f(x1)) /2
42 E5 = -h^2 * (fp(x1)-fp(x0)) /12
43 C5 = I5 + E5
44
45 n=64;
46 h=(x1-x0)/n;
47 I6 = ((x1-x0)/2) * (f(x0)+f(x1)) /2
48 E6 = -h^2 * (fp(x1)-fp(x0)) /12
49 C6 = I6 + E6

```

Scilab code Exa 5.4 Simpson s rule for integration

```

1          // PG (258)
2
3  deff(' [y]=f(x)', 'y=exp(x)*cos(x)')
4  x0=0;
5  xn=%pi;
6  x=x0:xn;
7
8  // True value
9
10 I = integrate('exp(x)*cos(x)', 'x', x0, x1)
11
12 // Using Simpson's rule
13
14 N=2;

```

```

15 h=(xn-x0)/N;
16 x1=x0+h;
17 x2=x0+2*h;
18     I1 = h*(f(x0)+4*f(x1)+f(x2))/3
19
20 N=4;
21 h=(xn-x0)/N;
22 x1=x0+h;
23 x2=x0+2*h;
24 x3=x0+3*h;
25 x4=x0+4*h;
26     I2 = h*(f(x0)+4*f(x1)+2*f(x2)+4*f(x3)+f(x4))/3
27
28 N=8;
29 h=(xn-x0)/N;
30 x1=x0+h;
31 x2=x0+2*h;
32 x3=x0+3*h;
33 x4=x0+4*h;
34 x5=x0+5*h;
35 x6=x0+6*h;
36 x7=x0+7*h;
37 x8=x0+8*h;
38     I3 = h*(f(x0)+4*f(x1)+2*f(x2)+4*f(x3)+2*f(x4)+4*
           f(x5)+2*f(x6)+4*f(x7)+f(x8))/3

```

Scilab code Exa 5.5 Trapezoidal and simpson integration

```

1           // PG (261)
2
3 // Example 1
4
5 def f (' [y]=f(x) ', 'y=x^(7/2) ')
6 def fp (' [y]=fp(x) ', 'y=3.5*x^(5/2) ')
7 def fpp (' [y]=fpp(x) ', 'y=8.75*x^(3/2) ')

```

```

8 def f(' [y]=fppp(x)', 'y=(105*sqrt(x))/8')
9 def f(' [y]=fpppp(x)', 'y=(105*x^(-0.5))/16')
10
11 x0=0;
12 x1=1;
13 x=x0:x1;
14
15 // True value
16 I = integrate('x^(7/2)', 'x', x0, x1)
17
18 // Using Trapezoidal rule
19
20 n=2;
21 h=(xn-x0)/n;
22 I1 = (xn-x0) * (f(x0)+f(xn)) /4;
23 E1 = -h^2 * (fp(xn)-fp(x0)) /12 // Error
24
25 n=4;
26 h=(xn-x0)/n;
27 I2 = (xn-x0) * (f(x0)+f(xn)) /4;
28 E2 = -h^2 * (fp(xn)-fp(x0)) /12 // Error
29
30 // Using Simpson's rule
31
32 N=2;
33 h=(xn-x0)/N;
34 x1=x0+h;
35 x2=x0+2*h;
36 I1 = h*(f(x0)+4*f(x1)+f(x2))/3
37 E1 = -h^4*(xn-x0)*fpppp(0.5)/180
38
39 N=4;
40 h=(xn-x0)/N;
41 x1=x0+h;
42 x2=x0+2*h;
43 x3=x0+3*h;
44 x4=x0+4*h;
45 I2 = h*(f(x0)+4*f(x1)+2*f(x2)+4*f(x3)+f(x4))/3

```

46 $E2 = -h^4*(xn-x0)*fpppp(0.5)/180$

Scilab code Exa 5.6 Newton Cotes formulae

```
1          //    PG (266)
2
3 //    Commonly used Newton Cotes formulae:-
4
5 //    n=1
6
7 //    h/2 * [f(a)+f(b)] - (h^3)*f''(e)/12-----
   Trapezoidal rule
8
9 //    n=2
10
11 //    h/3 * [f(a)+4*f((a+b)/2)+f(b)] - (h^5)*f^(4)(e
   )/90-----Simpson's rule
12
13 //    n=3
14
15 //    3*h/8 * [f(a)+3*f(a+h)+3*f(b-h)+f(b)] - (3*h
   ^5)*f^(4)(e)/80
16
17 //    n=4
18
19 //    2*h/45 * [7*f(a)+32*f(a+h)+12*f((a+b)/2)+32*f(
   b-h)+7*f(b)] - (8*h^7)*f^(7)(e)/945
```

check Appendix [AP 6](#) for dependency:

legendrepol.sce

Scilab code Exa 5.7 Gaussian Quadrature


```

1          // PG (277)
2
3  defff ('[y]=f(x)', 'y=exp(x)*cos(x)')
4  x0=0;
5  x1=%pi;
6
7
8  // True value
9  I = integrate('exp(x)*cos(x)', 'x', x0, x1)
10
11 // Using Gaussian Quadrature
12
13 // For n=2, w=1
14
15 n=2;
16 p = legendrepol(n, 'x')
17 xr = roots(p);
18 A = [];
19
20 for j = 1:2
21     pd = derivat(p)
22     A = [A 2/((1-xr(j))^2)*(horner(pd, xr(j)))^2)]
23 end
24
25 tr = ((x1-x0)/2.*xr)+((x1+x0)/2)

```

check Appendix [AP 6](#) for dependency:

legendrepol.sce

Scilab code Exa 5.8 Gaussian Legendre Quadrature

```

1          // PG (278)
2
3  defff ('[y]=f(x)', 'y=exp(-x^2)')
4  x0=0;

```

```

5 x1=1;
6
7
8 // True value
9 I = integrate('exp(-x^2)', 'x', x0, x1)
10
11 // Using Gaussian Quadrature
12
13 // For n=2, w=1
14
15 n=2;
16 p = legendrepol(n, 'x')
17 xr = roots(p);
18 A = [];
19
20 for j = 1:2
21     pd = derivat(p)
22     A = [A 2/((1-xr(j))^2)*(horner(pd, xr(j)))^2)]
23 end
24
25 tr = ((x1-x0)/2.*xr)+((x1+x0)/2);
26
27 s = ((x1-x0)/2)*f(tr)
28 I = s*A

```

Scilab code Exa 5.9 Integration

```

1 // PG (280)
2
3 I1 = integrate('sqrt(x)', 'x', 0, 1)
4
5 I2 = integrate('1/(1+(x-%pi)^2)', 'x', 0, 5)
6
7 I3 = integrate('exp(-x)*sin(50*x)', 'x', 0, 2*%pi)

```

Scilab code Exa 5.10 Simpson Integration error

```
1          // PG (292)
2
3  defff ('[y]=f(x)', 'y=x^(3/2)')
4  x0=0;
5  xn=1;
6  x=x0:xn;
7
8  // True value
9
10 I = integrate('x^(3/2)', 'x', 0, 1)
11
12 // Using Simpson's rule
13
14 N=2;
15 h=(xn-x0)/N;
16 x1=x0+h;
17 x2=x0+2*h;
18 I1 = h*(f(x0)+4*f(x1)+f(x2))/3
19 I-I1
20
21 N=4;
22 h=(xn-x0)/N;
23 x1=x0+h;
24 x2=x0+2*h;
25 x3=x0+3*h;
26 x4=x0+4*h;
27 I2 = h*(f(x0)+4*f(x1)+2*f(x2)+4*f(x3)+f(x4))/3
28 I-I2
29
30 N=8;
31 h=(xn-x0)/N;
32 x1=x0+h;
```

```

33 x2=x0+2*h;
34 x3=x0+3*h;
35 x4=x0+4*h;
36 x5=x0+5*h;
37 x6=x0+6*h;
38 x7=x0+7*h;
39 x8=x0+8*h;
40 I3 = h*(f(x0)+4*f(x1)+2*f(x2)+4*f(x3)+2*f(x4)+4*
        f(x5)+2*f(x6)+4*f(x7)+f(x8))/3
41 I-I3

```

check Appendix [AP 7](#) for dependency:

romberg.sce

Scilab code Exa 5.11 Romberg Integration

```

1          // PG (297)
2
3 def f(' [y]=f(x) ', 'y=exp(x)*cos(x) ')
4 a=0;
5 b=%pi;
6 h=1;
7
8 // True value
9
10 I = integrate('exp(x)*cos(x)', 'x', a, b)
11
12 // Using Romberg integration
13
14 Romberg(a, b, f, h)

```

Scilab code Exa 5.12 Adaptive simpson

```

1          // PG (302)
2
3  deff(' [y]=f(x) ', 'y=sqrt(x) ')
4  funcprot(0)
5  a=0;
6  b=1;
7
8  // True value
9
10 I = integrate('sqrt(x)', 'x', a, b)

```

Scilab code Exa 5.13 Integration

```

1          // PG (307)
2
3  deff(' [y]=f(x) ', 'y=sqrt(-log(x)) ')
4  funcprot(0)
5  a=0;
6  b=1;
7
8  // True value
9
10 I = integrate('sqrt(-log(x))', 'x', a, b)

```

Scilab code Exa 5.14 Integration

```

1          // PG (313)
2
3  deff(' [y]=f(x) ', 'y=(log(x))/(x+2) ')
4  funcprot(0)
5  a=0;
6  b=1;
7

```

```
8 // True value
9
10 I = integrate('log(x)/(x+2)', 'x', a, b)
```

Chapter 6

Numerical methods for ordinary differential equations

Scilab code Exa 6.1 1st order linear differential equation

```
1          // PG (334)
2
3 //      dy/dt=-y
4 function ydot=f(y,t),ydot=-y,
5 endfunction
6 y0=0;t0=0;t=0:1:%pi;
7 y=ode(y0,t0,t,f)
8 plot(t,y)
```

Scilab code Exa 6.4 Stability of solution

```
1          // PG (339)
2
3 //      dy/dx=100y -101*(%e)^(-x)
4 function ydox=f(x,y),ydox=100*y -101*(%e)^(-x),
5 endfunction
```

```

5 funcprot(0)
6 y0=1;
7 x0=0;
8 x=0:5;
9 y=ode(y0,x0,x,f)
10
11 //      Solution will be  $Y(x) = \exp(-x)$ 
12
13 //      For the perturbed problem,  $dy/dx = 100*y -$ 
14 //       $101*\exp(-x)$ ,  $y(0) = 1+e$ 
15 //      Solution will be  $Y(x;e) = \exp(-x) + e*\exp(100*$ 
16 //       $x)$ 
17 //      This rapidly departs from the true solution.

```

check Appendix [AP 2](#) for dependency:

euler.sce

Scilab code Exa 6.5 Euler method

```

1          //      PG (344)
2
3 //       $dy/dx = y$ 
4
5 //  $y'=f(x, t)$ 
6 def f(' [z]=f(x, y) ', ' z=y ');
7
8 // execute the function euler1 , so as to call it to
9 // evaluate the value of y,
10
11 [y,x] = Euler1(0.40,1,2.00,0.2,f) // h=0.2;
12
13 [y,x] = Euler1(0.40,1,2.00,0.1,f) // h=0.1;
14
15 [y,x] = Euler1(0.40,1,2.00,0.05,f) // h=0.05;

```



```

16
17 // True solution is
18 Y = exp(x)
19
20
21 // dy/dx = (1/(1+x^2)) - (2*y^2)
22
23 // y'=f(x,t)
24 def f(' [z]=f(x,y)', 'z=(1/(1+x^2)) - (2*y^2) ');
25
26 // execute the function euler1 , so as to call it to
    evaluate the value of y,
27
28
29
30 [y,x] = Euler1(0,0,2,0.2,f) // h=0.2;
31
32 [y,x] = Euler1(0,0,2,0.1,f) // h=0.1;
33
34 [y,x] = Euler1(0,0,2,0.05,f) // h=0.05;
35
36 // True solution is
37 Y = x/(1+x^2)

```

check Appendix [AP 2](#) for dependency:

euler.sce

Scilab code Exa 6.6 Euler

```

1 // PG (351)
2
3 // dy/dx = -y + 2 * cos(x)
4
5 def f(' [y]=g(x,y)', 'y=-y+2*cos(x) ')
6 y0=1;

```

```

7 x0=0;
8 xn=5;
9
10 // execute the function euler1 , so as to call it to
    evaluate the value of y,
11
12 [y,x] = Euler1(y0,x0,xn,0.04,g) // h = 0.04
13
14 [y,x] = Euler1(y0,x0,xn,0.02,g) // h = 0.02
15
16 [y,x] = Euler1(y0,x0,xn,0.01,g) // h = 0.01

```

Scilab code Exa 6.7 Asymptotic error analysis

```

1 // PG (354)
2
3 // dy/dx = -y
4
5 def f (' [z]=f(x,y) ', ' z=-y ')
6 y0=1;
7
8 // True solution is
9 Y = exp(-x)
10 // The equation for D(x) is
11 // D'(x) = -D(x) + 0.5*exp(-x)
12 // D(0) = 0
13 // The solution is
14 // D(x) = 0.5*x*exp(-x)

```

Scilab code Exa 6.9 Midpoint and trapezoidal method

```

1 // PG (357)
2

```

```

3 //      1. The mid-point method is defined by
4
5 //       $y(n+1) = y(n-1) + 2*h*f(xn,yn)$ -----n>=1
6
7 //      It is an explicit two-step method.
8
9
10 //      The trapezoidal method is defined by
11
12 //       $y(n+1) = yn + h*[f(xn,yn) + f(x(n+1),y(n+1))$ 
13 //      ]-----n>=0
14 //      It is an implicit one-step method.

```

check Appendix [AP 2](#) for dependency:

euler.sce

Scilab code Exa 6.10 Euler

```

1 //      PG (365)
2
3 deff ( ' [z]=g(x,y) ', ' z=-y ' )
4 [y,x] = Euler1(0.25,1,2.25,0.25,g)
5
6 //-----
7
8 deff ( ' [z]=f(x,y) ', ' z=x-y^2 ' )
9 [y,x] = Euler1(0.25,0,3.25,0.25,f)

```

check Appendix [AP 5](#) for dependency:

trapezoidal.sce

Scilab code Exa 6.11 Trapezoidal method

```
1          //    PG (372)
2
3  deff(' [y]=f(x,y) ', 'y=-y^2 ')
4  [x,y] = trapezoidal(1,1,5,1,f)
```

Scilab code Exa 6.16 Adams Moulton method

```
1          //    PG (389)
2
3  //    Using Adams–Moulton Formula
4
5  deff(' [z]=f(x,y) ', 'z=(1/(1+x^2))-2*y^2 ')
6  y0 = 0;
7
8  //    Solution is  $Y(x) = x/(1+x^2)$ 
9
10 function [y,x] = adamsmoulton4(y0,x0,xn,h,f)
11
12 //adamsmoulton4 4th order method solving ODE
13 // dy/dx = f(y,x), with initial
14 //conditions y=y0 at x=x0. The
15 //solution is obtained for x = [x0:h:xn]
16 //and returned in y
17
18 umaxAllowed = 1e+100;
19
20 x = [x0:h:xn]; y = zeros(x); n = length(y); y(1) =
    y0;
21 for j = 1:n-1
22 if j<3 then
23     k1=h*f(x(j),y(j));
24     k2=h*f(x(j)+h,y(j)+k1);
25     y(j+1) = y(j) + (k2+k1)/2;
```

```

26 end;
27
28 if j>=2 then
29     y(j+2) = y(j+1) + (h/12)*(23*f(x(j+1),y(j+1))
        )-16*f(x(j),y(j))+5*f(x(j-1),y(j-1)));
30 end;
31 end;
32 endfunction
33
34 adamsmoulton4(0,2.0,10.0,2.0,f)

```

check Appendix [AP 2](#) for dependency:

euler.sce

Scilab code Exa 6.21 Euler method

```

1 // PG (405)
2
3 def f(' [y]=f(x,y)', 'y=lamda*y+(1-lamda)*cos(x)-(1+
    lamda)*sin(x)')
4 lamda = -1;
5 [x,y]=Euler1(1,1,5,0.5,f)
6 lamda = -10;
7 [x,y]=Euler1(1,1,5,0.1,f)
8 lamda = -50;
9 [x,y]=Euler1(1,1,5,0.01,f)

```

check Appendix [AP 5](#) for dependency:

trapezoidal.sce

Scilab code Exa 6.24 Trapezoidal method

```

1          //    PG (409)
2
3  def f(' [y]=f(x,y) ', 'y=lamda*y+(1-lamda)*cos(x)-(1+
      lamda)*sin(x)')
4  lamda = -1;
5  [x,y]=trapezoidal(1,1,5,0.5,f)
6  lamda = -10;
7  [x,y]=trapezoidal(1,1,5,0.5,f)
8  lamda = -50;
9  [x,y]=trapezoidal(1,1,5,0.5,f)

```

check Appendix [AP 4](#) for dependency:

`bvpeigen.sce`

check Appendix [AP 3](#) for dependency:

`eigenvectors.sce`

Scilab code Exa 6.31 Boundary value problem

```

1          //    PG (434)
2
3  //    2-point linear Boundary value problem
4
5
6  //    Boundary value problems with eigenvalues -
      case:  $d^2y/dx^2 + \lambda y = 0$ 
7  //    subject to  $y(0) = 0, y(1) = 0$ , where  $\lambda$  is
      unknown.
8  //    The finite-difference approximation is:
9  //     $(y(i-1) - 2y(i) + y(i+1))) = -\lambda \Delta x^2 y(i), i =$ 
      2, 3, ..., n-1
10
11
12 [x,y,lam] = BVPeigen1(1,5)

```

Chapter 7

Linear Algebra

Scilab code Exa 7.1 Orthonormal basis

```
1          // PG (470)
2
3 u1 = [1/2, sqrt(3)/2]
4 u2 = [-sqrt(3)/2, 1/2]
5
6 // For a given vector x = (x1,x2), it can be
   written as
7 // x = alpha1*u1 + alpha2*u2
8 // alpha1 = (x1+x2*sqrt(3))/2
9 // alpha2 = (x2-x1*sqrt(3))/2
10
11 // (1,0) = (1/2)*u1 - (sqrt(3)/2)*u2
```

Scilab code Exa 7.2 Canonical forms

```
1          // PG (476)
2
3 A = [0.2 0.6 0; 1.6 -0.2 0; -1.6 1.2 3.0]
```

```

4 U = [0.6 0 -0.8;0.8 0 0.6;0 1.0 0]
5 Ustar = inv(U)
6 T = Ustar*A*U
7 trace(A)
8 lam =spec(A)'
9 lam1 = lam(1,1)
10 lam2 = lam(1,2)
11 lam3 = lam(1,3)
12 lam1 + lam2 + lam3
13
14 //      trace(A) = lam1 + lam2 + lam3
15
16 det(A)
17 lam1*lam2*lam3
18
19 //      det(A) = lam1 * lam2 * lam3

```

Scilab code Exa 7.3 Orthonormal eigen vectors

```

1 //      PG (477)
2
3 A = [2 1 0;1 3 1;0 1 2]
4 lam = spec(A)'
5 lam1 = lam(1,1)
6 lam2 = lam(1,2)
7 lam3 = lam(1,3)
8 //      Orthonormal Eigen vectors
9
10 u1 = (1/sqrt(3))*[1;-1;1]
11 u2 = (1/sqrt(2))*[1;0;-1]
12 u3 = (1/sqrt(6))*[1;2;1]

```

Scilab code Exa 7.4 Vector and matrix norms


```

1          // PG (481)
2
3 x = [1,0,-1,2]
4     // 1-norm
5 norm(x,1)
6     // 2-norm
7 norm(x,2)
8     // infinity norm
9 norm(x,'inf')
```

Scilab code Exa 7.5 Frobenious norm

```

1          // PG (484)
2
3 // A be n * n
4 // norm(A*x,2)
5 // norm(A*x,2) <= norm(A,'fro') * norm(x,2)
6 // norm(A*B,'fro') = norm(A,'fro') * norm(B,'fro
   ')
```

Scilab code Exa 7.6 Norm

```

1          // PG (489)
2
3 A = [1 -2;-3 4]
4 norm(A,1)
5 norm(A,2)
6 norm(A,'inf')
7 lam = spec(A)
8 r = max(abs(lam))
9     //r <= norm(A,2)
```

Scilab code Exa 7.7 Inverse exists

```
1          // PG (494)
2
3 A = [4 1 0 0;1 4 1 0;0 1 4 1;0 0 1 4]
4 B = A/4 - eye()
5 norm(B, 'inf')
6 // Let (I+B = C)
7 C = eye() + B
8 inv(C)
9 // Inverse of (I + B) exists
10 norm(C, 'inf')
11 // Inverse of A exists.
```

Chapter 8

Numerical solution of systems of linear equations

Scilab code Exa 8.2 LU decomposition

```
1      //      EXAMPLE (PG 512)
2
3  A = [1 2 1;2 2 3;-1 -3 0]           //
      Coefficient matrix
4  b = [0 3 2]'                       //      Right
      hand matrix
5  [l,u] = lu(A)
6      //      l is lower triangular matrix & u is upper
      triangular matrix
7  l*u
8  if(A==l*u)
9      disp('A = LU is verified')
10 end
11 det(A)
12 det(u)
13 if(det(A)==det(u))
14     disp('Determinant of A is equal to that of its
      upper triangular matrix')
15
```

16 // Product rule of determinants is verified

Scilab code Exa 8.4 LU decomposition

```
1 // EXAMPLE (PG 518)
2
3 // Row interchanges on A can be represented
  by premultiplication of A
4 // by an appropriate matrix E, to get EA.
5 // Then, Gaussian Elimination leads to LU =
  PA
6
7 A = [0.729 0.81 0.9;1 1 1;1.331 1.21 1.1] //
  Coefficient Matrix
8 b = [0.6867 0.8338 1.000]' //
  Right Hand Matrix
9 [L,U,E] = lu(A)
10 // L is lower triangular matrix(mxn)
11 // U is upper triangular matrix(mxmin(m,n))
12 // E is permutation matrix(min(m,n)xn)
13 Z=L*U
14
15 disp("LU = EA")
16 E
17
18 // The result EA is the matrix A with first ,
  rows 1 & 3 interchanged ,
19 // and then rows 2 & 3 interchanged.
20
21 // NOTE:-According to the book, P is replaced
  by E here.
```

Scilab code Exa 8.5 Choleski Decomposition

```

1      //      EXAMPLE (PG 526)
2
3      disp("Consider Hilbert matrix of order three")
4
5      n=3;          //      Order of the matrix
6      A=zeros(n,n); //      a symmetric positive definite
          real or complex matrix.
7      for i=1:n    //      Initializing 'for' loop
8          for j=1:n
9              A(i,j)=1/(i+j-1);
10         end
11     end          //End of 'for' loop
12     A
13     chol(A)      //      Choleski
          Decomposition
14     L=[chol(A)]' //      Lower Triangular
          Matrix
15
16     //      The square roots obtained here can be
          avoided using a slight modification.
17     //      We find a diagonal matrix D & a lower
          triangular matrix (L^~),
18     //      with 1s on the diagonal such that A = (L
          ^~) * D * (L^~)',
19
20
21     //      chol(A) uses only the diagonal and upper
          triangle of A.
22     //      The lower triangular is assumed to be the
          (complex conjugate) transpose of the upper
23     //

```

Scilab code Exa 8.6 LU decomposition

```

1      //      EXAMPLE (PG 529)

```

```

2
3 // Consider the coefficient matrix for spline
  interpolation
4
5
6 A = [2 1 0 0;1 4 1 0;0 1 4 1;0 0 1 2]
7 [l,u] = lu(A); // LU Decomposition
8 U = l' // Lower Triangular matrix
9 L = u' // Upper triangular matrix

```

Scilab code Exa 8.7 Error analysis

```

1 // EXAMPLE (PG 531)
2
3 // Consider the linear system
4
5 // 7*x1 + 10*x2 = b1
6 // 5*x1 + 7*x2 = b2
7
8 A = [7 10;5 7] // Coefficient matrix
9 inv(A) // Inverse matrix
10
11 // cond(A)1 // Condition matrix
12
13 norm(A,1)*norm(inv(A),1)
14
15 // cond(A)2 // Condition matrix
16
17 norm(A,2)*norm(inv(A),2)
18
19 // These condition numbers all suggest that
  the above system
20 // may be sensitive to changes in the right
  side b.
21

```

```

22     //      Consider the particular case
23
24     b = [1 0.7]';           //      Right hand matrix
25     x = A\b;               //      Solution matrix
26
27     //      Solution matrix
28
29     x1 = x(1,:);
30     x2 = x(2,:);
31
32     //      For the perturbed system, we solve for:
33
34     b = [1.01 0.69]';     //      Right hand matrix
35     x = A\b;               //      Solution matrix
36
37     //      Solution matrix
38
39     x1 = x(1,:);
40     x2 = x(2,:);
41
42     //      The relative changes in x are quite large
43     //      when compared with
44     //      the size of the relative changes in the
45     //      right side b.

```

Scilab code Exa 8.8 Residual correction method

```

1     //      EXAMPLE (PG 541)
2
3     //      Consider a Hilbert matrix of order 3
4
5     n=3;                   //      Order of the matrix
6     A=zeros(n,n);        //      a symmetric positive definite
7     //      real or complex matrix.
8     for i=1:n             //      Initializing 'for' loop

```

```

8     for j=1:n
9         A(i,j)=1/(i+j-1);
10    end
11 end          //      End of 'for' loop
12 A
13
14    //      Rounding off to 4 decimal places
15 A = A*10^4;
16 A = int(A);
17 A = A*10^(-4);
18 disp(A)          //      Final Solution
19
20 H = A          //      Here H denoted H bar as denoted
    in the text
21
22 b = [1 0 0]';
23 x = H\b
24
25    //      Rounding off to 3 decimal places
26 x = x*10^3;
27 x = int(x);
28 x = x*10^(-3);
29 disp(x)          //      Final Solution
30
31 //Now, using elimination with Partial Pivoting, we
    get the following answers
32
33 x0 = [8.968 -35.77 29.77]';
34
35    //      ro is Residual correction
36
37 r0 = b - A*x0
38
39    //      A*e0 = r0
40
41 e0 = inv(A)*r0
42
43 x1 = x0 + e0

```



```

44
45     //      Repeating the above operations , we can
           get the values of r1 , x2 , e1 ...
46     //      The vector x2 is accurate to 4 decimal
           digits .
47     //      Note that  $x_1 - x_0 = e_0$  is an accurate
           predictor of the error  $e_0$  in  $x_0$ .

```

Scilab code Exa 8.9 Residual correction method

```

1 //EXAMPLE (PG 544)
2
3 //A(e) = A0 + eB
4
5 A0=[2 1 0;1 2 1;0 1 2]
6 B=[0 1 1;-1 0 1;-1 -1 0]
7 //inv(A(e)) = C = inv(A0)
8 C=inv(A0)
9 b=[0 1 2] '
10 x=A0\b
11 r=b-A0*x

```

Scilab code Exa 8.10 Gauss Jacobi method

```

1 //      EXAMPLE (PG 547)
2
3 //      Gauss Jacobi Method
4
5 A = [10 3 1;2 -10 3;1 3 10] //
           Coefficient Matrix
6 b = [14 -5 14] ' //      Right
           hand matrix
7

```

```

8 x = [0 0 0] ' // Initial
    Gauss
9 d = diag(A) //
    Diagonal elements of matrix A
10 a11 = d(1,1)
11 a22 = d(2,1)
12 a33 = d(3,1)
13 D = [a11 0 0;0 a22 0;0 0 a33] //
    Diagonal matrix of A
14 [L,U] = lu(A) // L is lower triangular matrix, U
    is upper triangular matrix
15 H = -inv(D)*(L+U)
16 C = inv(D)*b
17
18 for(m=0:6) // Initialising 'for' loop for
    setting no of iterations to 6
19     x = H*x+C;
20     disp(x)
21     m=m+1;
22     x; // Solution
23     // Rounding off to 4 decimal places
24     x = x*10^4;
25     x = int(x);
26     x = x*10^(-4);
27     disp(x) // Final Solution
28
29 end

```

check Appendix [AP 1](#) for dependency:

gaussseidel.sce

Scilab code Exa 8.11 Gauss seidel method

```

1 //EXAMPLE (PG 549)
2

```

```

3      //Gauss Seidel Method
4
5      exec gaussseidel.sce
6      A = [10 3 1;2 -10 3;1 3 10]      //      Coefficient
           matrix
7      b = [14 -5 14]'                 //      Right hand
           matrix
8      x0 = [0 0 0]'                   //      Initial Gauss
9      gaussseidel(A,b,x0)             //      Calling
           function
10
11      //      End the problem

```

Scilab code Exa 8.13 Conjugate gradient method

```

1      //      EXAMPLE (PG 568)
2
3      A= [5 4 3 2 1;4 5 4 3 2;3 4 5 4 3;2 3 4 5 4;1 2 3 4
           5]      //      Matrix of order 5
4      //      Getting the eigenvalues
5
6      lam = spec(A)                   //      lamda = spectral
           radius of matrix A
7
8      max(lam)                        //      Largest eigenvalue
9      min(lam)                        //      Smallest eigen
           value
10
11      //      For the error bound given earlier on
           Pg 567
12
13      c = min(lam)/max(lam)
14
15      (1-sqrt(c))/(1+sqrt(c))
16

```

```
17 // For linear system, choose the following
    values of b
18
19 b = [7.9380 12.9763 17.3057 19.4332 18.4196]';
20
21 x = A\b; // Solution matrix
22
23 // Rounding off to 4 decimal places
24 x = x*10^4;
25 x = int(x);
26 x = x*10^(-4)
27 disp(x) // Final Solution
```

Chapter 9

The Matrix Eigenvalue Problem

Scilab code Exa 9.1 Eigenvalues

```
1           // EXAMPLE 590
2
3 A = [4 1 0;1 0 -1;1 1 -4]
4 [n,m] = size(A);
5
6 if m<>n then
7     error('eigenvectors - matrix A is not square');
8     abort;
9 end;
10
11 lam = spec(A)           //Eigenvalues of
    matrix A
```

Scilab code Exa 9.2 Eigen values and matrix norm

```
1           // PG 591
```

```

2
3 n = 4
4 A = [4 1 0 0;1 4 1 0;0 1 4 1;0 0 1 4]
5 lam = spec(A)
6
7 // Since A is symmetric, all eigen values are
  real.
8 // The radii are all 1 or 2.
9 // The centers of all the circles are 4.
10 // All eigen values must all lie in the interval
    [2,6]
11 // Since the eigen values of inv(A) are the
    reciprocals of those of A,
12 // 1/6 <= mu <= 1/2
13
14 // Let inv(A) = B
15
16 B=inv(A);
17 norm(B,2)
18 n
19 i = 1:n;
20 j = 1:n;
21
22 // for j~i
23 // r = sum(abs(B(i,j)))
24
25 // norm(B,2) = r(B) <= 0.5

```

Scilab code Exa 9.3 Bounds for perturbed eigen values

```

1 // PG 593
2
3 disp("Consider Hilbert matrix of order three")
4
5 n=3; // Order of the matrix

```

```

6 A=zeros(n,n);// a symmetric positive definite
  real or complex matrix.
7 for i=1:n // Initializing 'for' loop
8   for j=1:n
9     A(i,j)=1/(i+j-1);
10  end
11 end //End of 'for' loop
12 A
13
14 [n,m] = size(A)
15
16 if m<>n then
17   error('eigenvectors - matrix A is not square');
18   abort;
19 end;
20
21 lam = spec(A)' //Eigenvalues of
  matrix A
22
23 lam1 = lam(1,1)
24 lam2 = lam(1,2)
25 lam3 = lam(1,3)
26
27 // Rounding off to 4 decimal places
28
29 A = A*10^4;
30 A = int(A);
31 A = A*10^(-4);
32 disp(A) // Final Solution
33
34 lamr = spec(A)'
35
36 lamr1 = lamr(1,1)
37 lamr2 = lamr(1,2)
38 lamr3 = lamr(1,3)
39
40 // Errors
41

```

```

42 lam = lamr
43
44     //     Relative Errors
45
46 R1 = (lam1-lamr1)/lam1
47 R2 = (lam2-lamr2)/lam2
48 R3 = (lam3-lamr3)/lam3

```

Scilab code Exa 9.4 Eigenvalues of nonsymmetric matrix

```

1         //     PG 594
2
3 A = [101 -90;110 -98]
4 [n,m] = size(A)
5
6 if m<>n then
7     error('eigenvectors - matrix A is not square');
8     abort;
9 end;
10
11 lam = spec(A) '           //Eigenvalues of
12     matrix A
13
14     //     A+E = [101-e -90-e;110 -98]
15     //     Let e = 0.001
16 e = 0.001;
17     //     Let A+E = D
18 D = [101-e -90-e;110 -98]
19
20 [n,m] = size(D)
21
22 if m<>n then
23     error('eigenvectors - matrix D is not square');
24     abort;

```



```

25 end;
26 lam = spec(D)' //Eigenvalues of
    matrix A

```

Scilab code Exa 9.5 Stability of eigenvalues for nonsymmetric matrices

```

1 // PG 599
2
3 // e = 0.001
4 // From earlier example :
5 // eigen values of matrix A are 1 and 2. So
6 // ...
7 // inv(P)*A*P = [1 0;0 2]
8
9 A = [101 -90;110 -98]
10 B = [-1 -1;0 0]
11 // From the above equation , we get:
12
13 P = [9/sqrt(181) -10/sqrt(221);10/sqrt(181) -11/sqrt
    (221)]
14 inv(P)
15 K = norm(P)*norm(inv(P)) // K is condition
    number
16 u1 = P(:,1)
17 u2 = P(:,2)
18 Q = inv(P)
19 R = Q'
20 w1 = R(:,1)
21 w2 = R(:,2)
22 s1 = 1/norm(w1,2)
23 norm(B)
24
25 // abs(lam1(e) - lam1) <= sqrt(2)*e/0.005 + O(e
    ^2) = 283*e + O(e^2)

```

Scilab code Exa 9.7 Rate of convergence

```
1          //      (PG 607)
2
3  A = [1 2 3;2 3 4;3 4 5]
4  lam = spec(A) '
5  lam1 = lam(1,3)
6  lam2 = lam(1,1)
7  lam3 = lam(1,2)
8
9          //      Theoretical ratio of convergence
10
11 lam2/lam1
12
13 b = 0.5*(lam2+lam3)
14 B = A-b*eye(3,3)
15
16          //      Eigen values of A-bI are:
17
18 lamb = spec(B) '
19 lamb1 = lamb(1,3)
20 lamb2 = lamb(1,2)
21 lamb3 = lamb(1,1)
22
23          //      Ratio of convergence for the power method
                applied to A-bI will be:
24
25 lamb2/lamb1
26
27          //      This is less than half the magnitude of
                the original ratio.
```

Scilab code Exa 9.8 Rate of convergence after extrapolation

```
1          // PG (608)
2
3 A = [1 2 3;2 3 4;3 4 5]
4 lam = spec(A)' // Eigen values of A
5 lam1 = lam(1,3)
6 lam2 = lam(1,1)
7 lam3 = lam(1,2)
8
9 // Theoretical ratio of convergence
10
11 lam2/lam1
12
13 // After extrapolating, we get
14     lame1 = 9.6234814
15
16 // Error:
17 lam1-lame1
```

Scilab code Exa 9.9 Householder matrix

```
1          // PG (610)
2
3 w = [1/3 2/3 2/3]'
4 w1 = w(1,1)
5 w2 = w(2,1)
6 w3 = w(3,1)
7
8 U = [1-2*abs(w1)^2 -2*w1*w2' -2*w1*w3'; -2*w1'*w2
      1-2*abs(w2)^2 -2*w2*w3'; -2*w1'*w3 -2*w2'*w3 1-2*
      abs(w3)^2]
9 U
10 inv(U)
11 // U = inv(U)-----Hence, U is Hermitian
```

```
12 U*U
13 // U*U = I—————Hence, U is orthogonal
```

Scilab code Exa 9.11 QR factorisation

```
1 // PG (613)
2
3 A = [4 1 1;1 4 1;1 1 4]
4 w1 = [0.985599 0.119573 0.119573] '
5 P1 = eye() - 2*w1*w1 '
6 A2 = P1*A
7 w2 = [0 0.996393 0.0848572] '
8 P2 = eye() - 2*w2*w2 '
9 R = P2*A2
10 Q = P1*P2
11 Q*R
12
13 // A = Q * R
14
15 abs(det(A))
16 abs(det(Q)*det(R))
17
18 // |det(A)| = |det(Q)*det(R)| = |det(R)| = 54 (
    approx)
19
20 lam = spec(A) '
21 lam1 = lam(1,1)
22 lam2 = lam(1,2)
23 lam3 = lam(1,3)
24 lam1 * lam2 * lam3
25
26 // Product of eigen values also comes out to be
    54
```

Scilab code Exa 9.12 Tridiagonal Matrix

```
1          // PG (617)
2
3 A = [1 3 4;3 1 2;4 2 1]
4 w2 = [0 2/sqrt(5) 1/sqrt(5)]'
5 P1 = eye() - 2*w2*w2'
6 T = P1' * A * P1 // Tridiagonal matrix
```

Scilab code Exa 9.13 Planner Rotation Orthogonal Matrix

```
1          // PG (619)
2
3 x = %pi/4
4 R = [cos(x) 0 sin(x);0 1 0;-sin(x) 0 cos(x)]
5
6 // Planner Rotation Orthogonal Matrix
```

Scilab code Exa 9.14 Eigen values of a symmetric tridiagonal Matrix

```
1          // PG (620)
2
3 T = [2 1 0 0 0 0;1 2 1 0 0 0;0 1 2 1 0 0;0 0 1 2 1
      0;0 0 0 1 2 1;0 0 0 0 1 2]
4 lam = spec(T)'
5 lam1 = lam(1,1)
6 B = [2-lam1 1 0 0 0 0;1 2-lam1 1 0 0 0;0 1 2-lam1 1
      0 0;0 0 1 2-lam1 1 0;0 0 0 1 2-lam1 1;0 0 0 0 1
      2]
```

```
7 f0 = abs(det(B))
8 f1 = 2-lam1
```

Scilab code Exa 9.15 Sturm Sequence property

```
1          // PG (621)
2
3 // For the previous example, consider the
  sequence f0, f1....f6
4
5 // For lam = 3,
6
7 // (f0 ,..... f6) = (1, -1, 0, 1, -1, 0, 1)
8
9 // The corresponding sequence of signs is
10
11 // (+, -, +, +, -, +, +)
12
13 // and s(3) = 2
```

Scilab code Exa 9.16 QR Method

```
1          // PG (624)
2
3 A1 = [2 1 0; 1 3 1; 0 1 4]
4 lam = spec(A1)'
5 [Q1, R1] = qr(A1);
6 A2 = R1 * Q1
7 [Q2, R2] = qr(A2);
8 A3 = R2 * Q2
9 [Q3, R3] = qr(A3);
10 A4 = R3 * Q3
11 [Q4, R4] = qr(A4);
```

```

12 A5 = R4 * Q4
13 [Q5,R5] = qr(A5);
14 A6 = R5 * Q5
15 [Q6,R6] = qr(A6);
16 A7 = R6 * Q6
17 [Q7,R7] = qr(A7);
18 A8 = R7 * Q7
19 [Q8,R8] = qr(A8);
20 A9 = R8 * Q8
21 [Q9,R9] = qr(A9);
22 A10 = R9 * Q9
23 [Q10,R10] = qr(A10);

```

Scilab code Exa 9.18 Calculation of Eigen vectors and Inverse iteration

```

1          // PG (631)
2
3 A = [2 1 0;1 3 1;0 1 4]
4 lam = spec(A)
5 [L,U] = lu(A)
6 y1 = [1 1 1]'
7 w1 = [3385.2 -2477.3 908.20]'
8 z1 = [w1/norm(w1,'inf')]
9 w2 = [20345 -14894 5451.9]'
10 z2 = [w2/norm(w2,'inf')]
11 z3 = z2
12
13 // The true answer is
14
15 x3 = [1 1-sqrt(3) 2-sqrt(3)]'
16
17 // z2 equals x3 to within the limits of rounding
error accumulations.

```

Scilab code Exa 9.19 Inverse Iteration

```
1          // PG (633)
2
3 A = [2 1 0;1 3 1;0 1 4]
4 lam = spec(A)
5 [L,U] = lu(A)
6 y1 = [1 1 1]'
7 w1 = [3385.2 -2477.3 908.20]'
8 z1 = [w1/norm(w1,'inf')]
9 w2 = [20345 -14894 5451.9]'
10 z2 = [w2/norm(w2,'inf')]
11 z3 = z2
12
13 // The true answer is
14
15 x3 = [1 1-sqrt(3) 2-sqrt(3)]'
16
17 // z2 equals x3 to within the limits of rounding
    error accumulations.
18
19 // Consider lam = 1.2679
20
21 // 0.7321*x1 + x2 = 0
22 // x1 + 1.7321*x2 + x3 = 0
23 // Taking x1= 1.0, we have the approximate
    eigenvector
24
25 // x = [1.0000 -0.73210 0.26807]
26
27
28 // Compared with the true answer obtained above,
    this is a slightly poorer
29 // result obtained by inverse iteration.
```

Appendix

Scilab code AP 1 Gauss seidel method

```
1 function [x]=gaussseidel(A,b,x0)
2 [nA,mA]=size(A)
3 n=nA
4 [L,U] = lu(A)
5 d = diag(A)
6 a11 = d(1,1)
7 a22 = d(2,1)
8 a33 = d(3,1)
9 D = [a11 0 0;0 a22 0;0 0 a33]
10 H = -inv(L+D)*U
11 C = inv(L+D)*b
12 for m=0:3
13     x = -inv(D)*(L+U)*x + inv(D)*b
14     m=m+1
15     disp(x)
16 end
17
18 endfunction
```

Scilab code AP 2 Euler method

```
1 function [x,y] = Euler1(x0,y0,xn,h,g)
2
3 //Euler 1st order method solving ODE
4 // dy/dx = g(x,y), with initial
5 // conditions y=y0 at x = x0. The
```

```

6 //solution is obtained for x = [x0:h:xn]
7 //and returned in y
8
9 ymaxAllowed = 1e+100
10
11 x = [x0:h:xn];
12 y = zeros(x);
13 n = length(y);
14 y(1) = y0;
15
16 for j = 1:n-1
17     y(j+1) = y(j) + h*g(x(j),y(j));
18     if y(j+1) > ymaxAllowed then
19         disp('Euler 1 - WARNING: underflow or
20             overflow ');
21         disp('Solution sought in the following range:
22             ');
23         disp([x0 h xn]);
24         disp('Solution evaluated in the following
25             range: ');
26         disp([x0 h x(j)]);
27         n = j;
28         x = x(1,1:n); y = y(1,1:n);
29         break;
30     end;
31 end;
32 endfunction
33
34 //End function Euler1

```

Scilab code AP 3 Eigen vectors

```

1 function [x,lam] = eigenvectors(A)
2
3 //Calculates unit eigenvectors of matrix A
4 //returning a matrix x whose columns are
5 //the eigenvectors. The function also

```

```

6 //returns the eigenvalues of the matrix.
7
8 [n,m] = size(A);
9
10 if m<>n then
11     error('eigenvectors - matrix A is not square');
12     abort;
13 end;
14
15 lam = spec(A)'; //Eigenvalues of
    matrix A
16
17 x = [];
18
19 for k = 1:n
20     B = A - lam(k)*eye(n,n); //Characteristic matrix
21     C = B(1:n-1,1:n-1); //Coeff. matrix for
        reduced system
22     b = -B(1:n-1,n); //RHS vector for
        reduced system
23     y = C\b; //Solution for reduced system
24     y = [y;1]; //Complete eigenvector
25     y = y/norm(y); //Make unit eigenvector
26     x = [x y]; //Add eigenvector to matrix
27 end;
28
29 endfunction
30 //End of function

```

Scilab code AP 4 Boundary value problem

```

1 function [x,y,lam] = BVPeigen1(L,n)
2
3 Dx = L/(n-1);
4 x=[0:Dx:L];
5 a = 1/Dx^2;
6 k = n-2;
7

```

```

8 A = zeros(k,k);
9 for j = 1:k
10     A(j,j) = 2*a;
11 end;
12 for j = 1:k-1
13     A(j,j+1) = -a;
14     A(j+1,j) = -a;
15 end;
16
17 exec eigenvectors.sce
18
19 [yy,lam]=eigenvectors(A);
20 //disp('yy');disp(yy);
21
22 y = [zeros(1,k);yy;zeros(1,k)];
23 //disp('y');disp(y);
24
25
26 xmin=min(x);xmax=max(x);ymin=min(y);ymax=max(y);
27 rect = [xmin ymin xmax ymax];
28
29 if k>=5 then
30     m = 5;
31 else
32     m = k;
33 end
34
35
36 endfunction

```

Scilab code AP 5 Trapezoidal method

```

1 function [x,y] = trapezoidal(x0,y0,xn,h,g)
2
3 //Trapezoidal method solving ODE
4 // dy/dx = g(x,y), with initial
5 //conditions y=y0 at x = x0. The
6 //solution is obtained for x = [x0:h:xn]

```

```

7 //and returned in y
8
9 ymaxAllowed = 1e+100
10
11 x = [x0:h:xn];
12 y = zeros(x);
13 n = length(y);
14 y(1) = y0;
15
16 for j = 1:n-1
17     y(j+1) = y(j) + h*(g(x(j),y(j))+g(x(j+1),y(j+1)))
18         )/2;
19     if y(j+1) > ymaxAllowed then
20         disp('Euler 1 - WARNING: underflow or
21             overflow ');
22         disp('Solution sought in the following range:
23             ');
24         disp([x0 h xn]);
25         disp('Solution evaluated in the following
26             range: ');
27         disp([x0 h x(j)]);
28         n = j;
29         x = x(1,1:n); y = y(1,1:n);
30         break;
31     end;
32 end;
33 endfunction
34
35 //End function trapezoidal

```

Scilab code AP 6 Legendre Polynomial

```

1
2 function [pL] = legendrepol(n,var)
3
4 // Generates the Legendre polynomial
5 // of order n in variable var

```

```

6
7 if n == 0 then
8     cc = [1];
9 elseif n == 1 then
10    cc = [0 1];
11 else
12     if modulo(n,2) == 0 then
13         M = n/2
14     else
15         M = (n-1)/2
16     end;
17
18     cc = zeros(1,M+1);
19     for m = 0:M
20         k = n-2*m;
21         cc(k+1) = ...
22             (-1)^m*gamma(2*n-2*m+1)/(2^n*gamma(m+1)*
                gamma(n-m+1)*gamma(n-2*m+1));
23     end;
24 end;
25
26 pL = poly(cc,var,'coeff');
27
28 // End function legendrepol

```

Scilab code AP 7 Romberg Integration

```

1 function [I]=Romberg(a,b,f,h)
2
3 // This function calculates the numerical
  integral of f(x) between
4 // x = a and x = b, with intervals h.
  Intermediate results are obtained
5 // by using SCILAB's own intrtrap function
6
7 x=(a:h:b)
8 x1=x(1,1)
9 x2=x(1,2)

```

```

10 x3=x(1,3)
11 x4=x(1,4)
12 y1=f(x1)
13 y2=f(x2)
14 y3=f(x3)
15 y4=f(x4)
16 y=[y1 y2 y3 y4]
17 I1 = inttrap(x,y)
18 x=(a:h/2:b)
19 x1=x(1,1)
20 x2=x(1,2)
21 x3=x(1,3)
22 x4=x(1,4)
23 x5=x(1,5)
24 x6=x(1,6)
25 x7=x(1,7)
26 y1=f(x1)
27 y2=f(x2)
28 y3=f(x3)
29 y4=f(x4)
30 y5=f(x5)
31 y6=f(x6)
32 y7=f(x7)
33 y=[y1 y2 y3 y4 y5 y6 y7]
34 I2 = inttrap(x,y)
35 I = I2 + (1.0/3.0)*(I2-I1)
36
37 endfunction
38 //end function Romberg

```

Scilab code AP 8 Lagrange

```

1 function [P]=lagrange(X,Y)
2
3     // X nodes ,Y values
4     // P is the numerical Lagrange polynomial
5     // interpolation
6     n=length(X)

```

```

6      //      n is the number of nodes. (n-1) is the
           degree
7  x=poly(0,"x")
8  P=0
9  for i=1:n, L=1
10     for j=[1:i-1,i+1:n] L=L*(x-X(j))/(X(i)-X(j))
11         end
12  P=P+L*Y(i)
13  end
14  endfunction

```

Scilab code AP 9 Muller method

```

1  function x=muller(x0,x1,x2,f)
2      R=3;
3      PE=10^-8;
4      maxval=10^4;
5      for n=1:1:R
6
7          La=(x2-x1)/(x1-x0);
8          Da=1+La;
9          ga=La^2*f(x0)-Da^2*f(x1)+(La+Da)*f(x2);
10         Ca=La*(La*f(x0)-Da*f(x1)+f(x2));
11
12         q=ga^2-4*Da*Ca*f(x2);
13         if q<0 then q=0;
14         end
15         p= sqrt(q);
16         if ga<0 then p=-p;
17         end
18         La=-2*Da*f(x2)/(ga+p);
19         x=x2+(x2-x1)*La;
20         if abs(f(x))<=PE then break
21         end
22         if (abs(f(x))>maxval) then error('Solution
           diverges ');
23         abort;
24         break

```



```

25         else
26         x0=x1;
27         x1=x2;
28         x2=x;
29         end
30     end
31     disp(n," no. of iterations =")
32 endfunction

```

Scilab code AP 10 Secant method

```

1 function [x]=secant(a,b,f)
2     N=100; // define max. no. iterations
           // to be performed
3     PE=10^-4 // define tolerance for
           // convergence
4     for n=1:1:N // initiating for loop
5         x=a-(a-b)*f(a)/(f(a)-f(b));
6         if abs(f(x))<=PE then break; //checking for
           // the required condition
7         else a=b;
8             b=x;
9         end
10    end
11    disp(n," no. of iterations =") //
12 endfunction

```

Scilab code AP 11 Newton

```

1 function x=newton(x,f,fp)
2     R=100;
3     PE=10^-8;
4     maxval=10^4;
5
6     for n=1:1:R
7         x=x-f(x)/fp(x);
8         if abs(f(x))<=PE then break
9         end

```

```

10         if (abs(f(x))>maxval) then error('Solution
           diverges ');
11             abort
12             break
13         end
14     end
15     disp(n," no. of iterations =")
16 endfunction

```

Scilab code AP 12 Aitken1

```

1 // this program is exclusively coded to perform one
  iteration of aitken method,
2
3 function x0aa=aitken(x0,x1,x2,g)
4 x0a=x0-(x1-x0)^2/(x2-2*x1+x0);
5 x1a=g(x0a);
6 x2a=g(x1a);
7 x0aa=x0a-(x1a-x0a)^2/(x2a-2*x1a+x0a);
8
9 endfunction

```

Scilab code AP 13 Bisection method

```

1 function x=bisection(a,b,f)
2     N=100; //
   define max. number of iterations
3     PE=10^-4 //
   define tolerance
4     if (f(a)*f(b) > 0) then
5         error('no root possible f(a)*f(b) > 0')
           // checking if the decided range is
           containing a root
6         abort;
7     end;
8     if(abs(f(a)) <PE) then
9         error('solution at a') //
           seeing if there is an approximate root

```

```

        at a,
10         abort;
11     end;
12     if(abs(f(b)) < PE) then //
        seeing if there is an approximate root at b,
13     error('solution at b')
14     abort;
15     end;
16     x=(a+b)/2
17     for n=1:1:N //
        initialising 'for' loop,
18         p=f(a)*f(x)
19         if p<0 then b=x ,x=(a+x)/2;
            //checking for the required conditions( f
            (x)*f(a)<0),
20         else
21             a=x
22             x=(x+b)/2;
23         end
24         if abs(f(x))<=PE then break
            // instruction to come out of the loop
            after the required condition is achieved,
25         end
26     end
27     disp(n," no. of iterations =")
        // display the no. of iterations took to
        achive required condition,
28 endfunction

```
