

Scilab Textbook Companion for  
Linear Algebra And Its Applications  
by G. Strang<sup>1</sup>

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# Book Description

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Scilab numbering policy used in this document and the relation to the above book.

**Exa** Example (Solved example)

**Eqn** Equation (Particular equation of the above book)

**AP** Appendix to Example(Scilab Code that is an Appednix to a particular Example of the above book)

For example, Exa 3.51 means solved example 3.51 of this book. Sec 2.3 means a scilab code whose theory is explained in Section 2.3 of the book.

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# Chapter 1

## Matrix Notation and Matrix Multiplication

Scilab code Exa 1.3.1 Breakdown of elimination

```
1 clear;
2 close;
3 clc;
4 a =[1 1 1;2 2 5;4 6 8]
5 disp('x=[u;v;w]')
6 disp('R2=R2-R1,R3=R3-4*R1')
7 a(2,:)=a(2,)-2*a(1,);
8 a(3,:)=a(3,)-4*a(1,);
9 disp(a);
10 disp('R2<->R3')
11 b=a(2,);
12 a(2,:)=a(3,);
13 a(3,:)=b;
14 disp(a);
15 disp('The system is now triangular and the equations
      can be solved by Back substitution');
16 //end
```

---



### Scilab code Exa 1.3.2 Breakdown of elimination

```
1 clear ;
2 close ;
3 clc ;
4 a =[1 1 1;2 2 5;4 4 8];
5 disp(a, 'a=');
6 disp('x=[u;v;w] ');
7 disp('R2=R2-2*R1, R3=R3-4*R1 ');
8 a(2,:)=a(2,:)-2*a(1,:);
9 a(3,:)=a(3,:)-4*a(1,:);
10 disp(a);
11 disp('No exchange of equations can avoid zero in the
      second pivot position ,therefore the equations
      are unsolvable ');
12 //end
```

---

### Scilab code Exa 1.4.1 Multiplication of Two Matrices

```
1 clear ;
2 close ;
3 clc ;
4 A=[2 3;4 0];
5 disp(A, 'A=');
6 B=[1 2 0;5 -1 0];
7 disp(B, 'B');
8 disp(A*B, 'AB=')
9 //end
```

---

#### Scilab code Exa 1.4.2 Multiplication with Row exchange matrix

```
1 clear;
2 close;
3 clc;
4 A=[2 3;7 8];
5 disp(A, 'A=');
6 P=[0 1;1 0];
7 disp(P, 'P(Row exchange matrix)=')
8 disp(P*A, 'PA=')
9 //end
```

---

#### Scilab code Exa 1.4.3 Multiplication with Identity Matrix

```
1 //page 24
2 clear;
3 close;
4 clc;
5 A=[1 2;3 4];
6 disp(A, 'A=');
7 I=eye(2,2);
8 disp(I, 'I=');
9 disp(I*A, 'IA=')
10 //end
```

---

#### Scilab code Exa 1.4.4 Matrix multiplication not commutative

```
1 //page 25
2 clear;
3 close;
4 clc;
5 E=eye(3,3);
6 E(2,:) = E(2,:) - 2*E(1,:);
```

```

7  disp(E, 'E=');
8  F=eye(3,3);
9  F(3,:)=F(3,:)+F(1,:);
10 disp(F, 'F=');
11 disp(E*F, 'EF=')
12 disp(F*E, 'FE=')
13 disp('Here ,EF=FE,so this shows that these two
      matrices commute')
14 //end

```

---

#### Scilab code Exa 1.4.5 Order of Elimination

```

1  //page 25
2  clear;
3  close;
4  clc;
5  E=eye(3,3);
6  E(2,:)=E(2,:)-2*E(1,:);
7  disp(E, 'E')
8  F=eye(3,3);
9  F(3,:)=F(3,:)+F(1,:);
10 disp(F, 'F=');
11 G=eye(3,3);
12 G(3,:)=G(3,:)+G(2,:);
13 disp(G, 'G')
14 disp(G*E, 'GE=')
15 disp(E*G, 'EG=')
16 disp('Here EG is not equal to GE,Therefore these two
      matrices do not commute and shows that most
      matrices do not commute.')
17 disp(G*F*E, 'GFE=')
18 disp(E*F*G, 'EFG=')
19 disp('The product GFE is the true order of elimination
      .It is the matrix that takes the original A to
      the upper triangular U.')

```

20 //end

---

### Scilab code Exa 1.5.1 Triangular factorization

```
1 //page 34
2 clear;
3 close;
4 clc;
5 A=[1 2;3 8];
6 disp(A, 'A=');
7 [L,U]=lu(A);
8 disp(L, 'L=');
9 disp(U, 'U=');
10 disp('LU=')
11 disp(L*U)
12 disp('This shows that LU=A')
13 //end
```

---

### Scilab code Exa 1.5.2 To check LU equals to A

```
1 //page 34
2 clear;
3 close;
4 clc;
5 A=[0 2;3 4];
6 disp(A, 'A=')
7 disp('Here this cannot be factored into A=LU,(Needs
      a row exchange)');
8 //end
```

---

**Scilab code Exa 1.5.3** To check LU equals to A

```
1 //page 34
2 clear;
3 close;
4 clc;
5 disp('Given Matrix:')
6 A=[1 1 1;1 2 2;1 2 3];
7 disp(A, 'A=');
8 [L,U]=lu(A);
9 disp(L, 'L=');
10 disp(U, 'U=');
11 disp(L*U, 'LU=');
12 disp('Here LU=A,from A to U there are subtraction of
      rows.Frow U to A there are additions of rows');
13 //end
```

---

**Scilab code Exa 1.5.4** If U equals to I then L equals to A

```
1 //page 34
2 clear;
3 close;
4 clc;
5 a=rand(1);
6 b=rand(1);
7 c=rand(1);
8 L=[1 0 0;a 1 0;b c 1];
9 disp(L, 'L=');
10 U=eye(3,3);
11 disp(U, 'U=');
12 E=[1 0 0;-a 1 0;0 0 1];
13 disp(E, 'E=');
14 F=[1 0 0;0 1 0;-b 0 1];
15 disp(F, 'F=');
16 G=[1 0 0;0 1 0;0 -c 1];
```

```

17 disp(G, 'G=');
18 disp('A=inv(E)*inv(F)*inv(G)*U')
19 A=inv(E)*inv(F)*inv(G)*U;
20 disp(A, 'A=');
21 disp('When U is identity matrix then L is same as A'
      ');
22 //end

```

---

#### Scilab code Exa 1.5.5 Spilting A to L and U

```

1 //page 39
2 clear;
3 close;
4 clc;
5 A=[1 -1 0 0 ; -1 2 -1 0; 0 -1 2 -1; 0 0 -1 2];
6 disp(A, 'A=');
7 [L,U]=lu(A);
8 disp(U, 'U=');
9 disp(L, 'L=');
10 disp('This shows how a matrix A with 3 diagnols has
      factors L and U with two diagnols.')
11 //end

```

---

#### Scilab code Exa 1.5.6 Solving for X using L and U

```

1 //page 36
2 clear;
3 close;
4 clc;
5 a=[1 -1 0 0; -1 2 -1 0; 0 -1 2 -1; 0 0 -1 2];
6 disp(a, 'a=')
7 b=[1; 1; 1; 1]
8 disp(b, 'b=')

```

```

9  disp('Given Equation ,ax=b')
10 [L,U]=lu(a);
11  disp(U, 'U=');
12  disp(L, 'L=');
13  disp('Augmented Matrix of L and b=');
14  A=[L b];
15  disp(A)
16  c=zeros(4,1);
17  n=4;
18  //Algorithm Finding the value of c
19  c(1)=A(1,n+1)/A(1,1);
20  for i=2:n;
21      sumk=0;
22      for k=1:n-1
23          sumk=sumk+A(i,k)*c(k);
24      end
25      c(i)=(A(i,n+1)-sumk)/A(i,i)
26  end
27  disp(c, 'c=');
28  x=zeros(4,1);
29  disp('Augmented matrix of U and c=')
30  B=[U c];
31  disp(B)
32  //Algorithm for finding value of x
33  x(n)=B(n,n+1)/B(n,n);
34  for i=n-1:-1:1;
35      sumk=0;
36      for k=i+1:n
37          sumk=sumk+B(i,k)*x(k);
38      end
39      x(i)=(B(i,n+1)-sumk)/B(i,i);
40  end
41  disp(x, 'x=')
42  //end

```

---

### Scilab code Exa 1.5.7 Elimination in a nutshell

```
1 //page 39
2 clear;
3 close;
4 clc;
5 A=[1 1 1;1 1 3;2 5 8];
6 disp(A, 'A=');
7 [L,U,P]=lu(A);
8 disp(L, 'L=');
9 disp(U, 'U=');
10 disp(P, 'P=');
11 disp(P*A, 'PA=')
12 disp(L*U, 'LU=')
13 disp('This shows that PA is the same as LU')
14 //end
```

---

### Scilab code Exa 1.6.1 Gauss Jordan method

```
1 //page 47
2 clear;
3 close;
4 clc;
5 disp('Given matrix:')
6 A=[2 1 1;4 -6 0;-2 7 2];
7 disp(A);
8 [n,m]=size(A);
9 disp('Augmented matrix :')
10 a=[A eye(n,m)];
11 disp(a)
12 disp('R2=R2-2*R1, R3=R3-(-2)*R1');
13 a(2,:)=a(2,:)-2*a(1,:);
14 a(3,:)=a(3,:)-(-1)*a(1,:);
15 disp(a)
16 disp('R3=R3-(-1)*R2');
```



```

17 a(3,:) = a(3,:) - (-1)*a(2,:);
18 disp(a, 'a=')
19 disp(a, '[U inv(L)] :')
20 disp('R2=R2-(-2)*R3, R1=R1-R3')
21 a(2,:) = a(2,:) - (-2)*a(3,:);
22 a(1,:) = a(1,:) - a(3,:);
23 disp(a)
24 disp('R1=R1-(-1/8)*R2')
25 a(1,:) = a(1,:) - (-1/8)*a(2,:);
26 disp(a)
27 a(1,:) = a(1, :)/a(1, 1);
28 a(2,:) = a(2, :)/a(2, 2);
29 disp(' [I inv(A)] :')
30 a(3,:) = a(3, :)/a(3, 3);
31 disp(a);
32 disp('inv(A) :')
33 a(:, 4:6);
34 disp(a(:, 4:6))

```

---

### Scilab code Exa 1.6.2 Symmetric products

```

1 //Caption :Symmetric Products
2 //Example:1.6.2 -To Find the product of transpose(R)
  and R.
3 //page 51
4 clear;
5 close;
6 clc;
7 R=[1 2];
8 disp(R, 'R=');
9 Rt=R';
10 disp(Rt, 'Transpose of the given matrix is :')
11 disp(R*Rt, 'The product of R & transpose(R) is :')
12 disp(Rt*R, 'The product of transpose(R)& R is :')
13 disp('Rt*R and R*Rt are not likely to be equal even

```

```
    if m==n. ')
14 //end
```

---

# Chapter 2

## Vector Spaces

Scilab code Exa 2.1.1 Vector Spaces and subspaces

```
1 //page 70
2 clear;
3 close;
4 clc;
5 disp('Consider all vectors in R^2 whose components
      are positive or zero')
6 disp('The subset is first Quadrant of x-y plane ,the
      co-ordinates satisfy x>=0 and y>=0.It is not a
      subspace.')
```

---

```
7 v=[1,1];
8 disp(v,'If the Vector=');
9 disp('Taking a scalar ,c=-1')
10 c=-1; //scalar
11 disp(c*v,'c*v=')
12 disp('It lies in third Quadrant instead of first ,
      Hence violating the rule(ii).')
```

---

```
13 //end
```

Scilab code Exa 2.1.2 Vector Spaces and subspaces

```

1 //page 71
2 clear;
3 close;
4 clc;
5 disp('Take vector space of 3X3 matrices')
6 disp('One possible subspace is the set of lower
    triangular matrices ,Another is set of symmetric
    matrices ')
7 disp('A+B,cA are both lower triangular if A and B
    are lower triangular ,and are symmetric if A and
    B are symmetric and Zero matrix is in both
    subspaces ')

```

---

#### Scilab code Exa 2.3.1 Linear Independence

```

1 //page 92
2 clear;
3 close;
4 clc;
5 disp('For linear independence , $C_1V_1+C_2V_2+\dots+C_kV_k=0$ 
    ')
6 disp('If we choose  $V_1$ =zero vector ,then the set is
    linearly dependent.We may choose  $C_1=3$  and all
    other  $C_i=0$ ;this is a non-trivial solution that
    produces zero. ')
7 //end

```

---

#### Scilab code Exa 2.3.2 Linear Independence

```

1 //page 92
2 clear;
3 close;
4 clc;

```

```

5 A=[1 3 3 2;2 6 9 5;-1 -3 3 0];
6 disp('Given matrix:')
7 disp(A)
8 B=A;
9 disp('C2->C2-3*C1')
10 A(:,2)=A(:,2)-3*A(:,1);
11 disp(A)
12 disp('Here ,C2=3*C1, Therefore the columns are
        linearly dependent.')
```

---

### Scilab code Exa 2.3.3 Linear Independence

```

1 clear;
2 close;
3 clc;
4 A=[3 4 2;0 1 5;0 0 2];
5 disp(A, 'A=');
6 disp('The columns of the triangular matrix are
        linearly independent, it has no zeros on the
        diagonal');
7 //end
```

---

### Scilab code Exa 2.3.4 Linear Independence

```

1 //page 93
2 clear;
3 close;
```

```

4 clc;
5 disp('The columns of the nxn identity matrix are
        independent. ')
6 n=input('Enter n:');
7 I=eye(n,n);
8 disp(I, 'I=');
9 //end

```

---

### Scilab code Exa 2.3.5 Linear Independence

```

1 //page 93
2 clear;
3 close;
4 clc;
5 disp('Three columns in R2 cannot be independent. ')
6 A=[1 2 1;1 2 3];
7 disp(A, 'Given matrix: ')
8 [L,U]=lu(A);
9 disp(U, 'U=');
10 disp('If c3 is 1 ,then back-substitution  $Uc=0$  gives
         $c2=-1,c1=1$ ,With these three weights ,the first
        column minus the second plus the third equals
        zero ,therefore linearly dependent. ')

```

---

### Scilab code Exa 2.3.6 Linear Independence

```

1 //page 93
2 clear;
3 close;
4 clc;
5 disp('The vectors  $w1=(1,0,0)$  , $w2=(0,1,0)$  , $w3=(-2,0,0)$ 
        span a plane (x-y plane) in R3. The first two

```

```
        vectors also span this plane , whereas w1 and w3
        span only a line. ');
6 //end
```

---

### Scilab code Exa 2.3.7 Linear Independence

```
1 //page 93
2 clear;
3 close;
4 clc;
5 disp('The column space of A is excatly the space
        that is spanned by its columns.The row space is
        spanned by the rows.The definition is made to
        order.Multiplying A by any x gives a combination
        of columns; it is a vector Ax in the column space
        . The coordinate vectors e_1 ,....e_n coming from
        the identity matrix span Rn. Every vector b=(b_1
        ..... ,b_n) is a combination of those columns.In
        this example the weights are the components b_i
        themselves:b=b_1e_1+.....+b_ne_n.But the columns
        of other matrices also span R_.'')
6 //end
```

---

### Scilab code Exa 2.3.8 Basis for a vector space

```
1 //page 93
2 clear;
3 close;
4 clc;
5 disp('Here, the vector v1 by itself is linearly
        independent , but it fails to span R2.The three
        vectors v1,v2,v3 certainly span R2, but are not
        independent. Any two of these vectors say v1 and
```

```
    v2 have both properties –they span and they are
    independent. So they form a basis. (A vector space
    does not have a unique basis)')
6 //end
```

---

### Scilab code Exa 2.3.9 Basis for a vector space

```
1 //page 96
2 clear;
3 close;
4 clc;
5 disp('These four columns span the column space U, but
    they are not independent.')
```

```
6 U=[1 3 3 2;0 0 3 1;0 0 0 0];
7 disp(U, 'U=');
8 disp('The columns that contains pivots (here 1st & 3
    rd) are a basis for the column space. These
    columns are independent, and it is easy to see
    that they span the space. In fact, the column space
    of U is just the x–y plane within R3. C(U) is
    not the same as the column space C(A) before
    elimination –but the number of independent columns
    did not change.')
```

---

### Scilab code Exa 2.4.1 The four fundamental subspaces

```
1 //page 107
2 clear;
3 close;
4 clc;
5 A=[1 2;3 6];
6 disp(A, 'A=');
7 [m,n]=size(A);
```



```

8  disp(m, 'm=');
9  disp(n, 'n=');
10 [v,pivot]=rref(A);
11 r=length(pivot);
12 disp(r, 'rank=')
13 cs=A(:,pivot);
14 disp(cs, 'Column space=');
15 ns=kernel(A);
16 disp(ns, 'Null space=');
17 rs=v(1:r,:)' ;
18 disp(rs, 'Row space=')
19 lns=kernel(A');
20 disp(lns, 'Left null sapce=');

```

---

#### Scilab code Exa 2.4.2 Inverse of a mxn matrix

```

1  //page 108
2  clear;
3  close;
4  clc;
5  A=[4 0 0;0 5 0];
6  disp(A, 'A=');
7  [m,n]=size(A);
8  disp(m, 'm=');
9  disp(n, 'n=')
10 r=rank(A);
11 disp(r, 'rank=');
12 disp('since m=r=2 ,there exists a right inverse .');
13 C=A'*inv(A*A');
14 disp(C, 'Best right inverse=')
15 //end

```

---

#### Scilab code Exa 2.5.1 Networks and discrete applied mathematics

```

1 //page 121
2 clear;
3 close;
4 clc;
5 disp('Applying current law A'y=f at nodes 1,2,3:')
6 A=[-1 1 0;0 -1 1; -1 0 1;0 0 -1;-1 0 0];
7 disp(A', 'A''=');
8 C=diag(rand(5,1)); //Taking some values for the
    resistances.
9 b=zeros(5,1);
10 b(3,1)=rand(1); //Taking some value of the battery.
11 f=zeros(3,1);
12 f(2,1)=rand(1); //Taking some value of the current
    source.
13 B=[b;f];
14 disp('The other equation is inv(C)y+Ax=b.The block
    form of the two equations is:')
15 C=[inv(C) A;A' zeros(3,3)];
16 disp(C);
17 X=['y1'; 'y2'; 'y3'; 'y4'; 'y5'; 'x1'; 'x2'; 'x3'];
18 disp(X, 'X=')
19 X=C\B;
20 disp(X, 'X=');
21 //end

```

---

# Chapter 3

## Orthogonality

Scilab code Exa 3.1.1 Orthogonal vectors

```
1 //page 143
2 clear;
3 close;
4 clc;
5 x1=[2;2;-1];
6 disp(x1,'x1=');
7 x2=[-1;2;2];
8 disp(x2,'x2=');
9 disp(x1'*x2,'x1'*x2=');
10 disp('Therefore,X1 is orthogonal to x2 .Both have
      length of sqrt(14).')
```

---

Scilab code Exa 3.1.2 Orthogonal vectors

```
1 //page 144
2 clear;
3 close;
4 clc;
```

```
5 disp('Suppose V is a plane spanned by v1=(1,0,0,0)
    and v2=(1,1,0,0). If W is the line spanned by w
    =(0,0,4,5), then w is orthogonal to both v's. The
    line W will be orthogonal to the whole plane V.')
```

---

### Scilab code Exa 3.1.3 Orthogonal vectors

```
1 //page 145
2 clear;
3 close;
4 clc;
5 A=[1 3;2 6;3 9];
6 disp(A, 'A=');
7 ns=kernel(A);
8 disp(ns, 'Null space=');
9 disp(A(1,:)*ns, 'A(1,:)*ns=');
10 disp(A(2,:)*ns, 'A(2,:)*ns=');
11 disp(A(3,:)*ns, 'A(3,:)*ns=');
12 disp('This shows that the null space of A is
    orthogonal to the row space.');
```

---

### Scilab code Exa 3.2.1 Projections onto a line

```
1 //page 155
2 clear;
3 close;
4 clc;
5 b=[1;2;3];
6 disp(b, 'b=');
7 a=[1;1;1];
8 disp(a, 'a=')
9 x=(a'*b)/(a'*a)
```

```

10 disp(x*a, 'Projection p of b onto the line through a
    is x^*a=');
11 disp((a'*b)/(sqrt(a'*a)*sqrt(b'*b)), 'cos(theta)=');
12 //end

```

---

### Scilab code Exa 3.2.2 Projections onto a line

```

1 //page 156
2 clear;
3 close;
4 clc;
5 a=[1;1;1];
6 disp(a, 'a=');
7 P=(a*a')/(a'*a);
8 disp(P, 'Matrix that projects onto a line through a
    =(1,1,1) is ');
9 //end

```

---

### Scilab code Exa 3.2.3 Projections onto a line

```

1 //page 156
2 clear;
3 close;
4 clc;
5 theta=45; //Taking some value for theta
6 a=[cos(theta);sin(theta)];
7 disp(a, 'a=');
8 P=(a*a')/(a'*a);
9 disp(P, 'Projection of line onto the theta-direction
    (theta taken as 45) in the x-y plane passing
    through a is ');
10 //end

```

---

### Scilab code Exa 3.3.1 Projection matrices

```
1 //page 165
2 clear;
3 close;
4 clc;
5 A=rand(4,4);
6 disp(A, 'A=');
7 P=A*inv(A'*A)*A';
8 disp('P=A*inv(A'*A)*A');
9 disp(P, 'Projection of a invertible 4x4 matrix on to
    the whole space is:');
10 disp('Its identity matrix.')
```

---

### Scilab code Exa 3.3.2 Least squares fitting of data

```
1 //page 166
2 clear;
3 close;
4 clc;
5 disp('b=C+Dt');
6 disp('Ax=b');
7 A=[1 -1;1 1;1 2];
8 disp(A, 'A=');
9 b=[1;1;3];
10 disp(b, 'b=');
11 disp('If Ax=b could be solved then they would be no
    errors, they can't be solved because the points
    are not on a line. Therefore they are solved by
    least squares.')
```

---

```
12 disp('so, A''Ax^=A''b');
```

```

13 x=zeros(1,2);
14 x=(A'*A)\(A'*b);
15 disp(x(1,1), 'C^ =');
16 disp(x(2,1), 'D^=');
17 disp('The best line is 9/7+4/7t')
18 //end

```

---

#### Scilab code Exa 3.4.1 Orthogonal matrices

```

1 //page 175
2 clear;
3 close;
4 clc;
5 theta=45;//Taking some value for theta.
6 Q=[cos(theta) -sin(theta);sin(theta) cos(theta)
    ];
7 disp(Q, 'Q=');
8 disp(Q', 'Q''=inv(Q)=');
9 disp('Q rotates every vector through an angle theta
    , and Q'' rotates it back through -theta.The
    columns are clearly orthogonal and they are
    orthonormal because sin^2(theta)+cos^2(theta)=1.
    ');
10 //end

```

---

#### Scilab code Exa 3.4.2 Orthogonal matrices

```

1 //page 175
2 clear;
3 close;
4 clc;
5 disp('Any permutation matrix is an orthogonal matrix
    .The columns are certainly unit vectors and

```

certainly orthogonal—because the 1 appears in a different place in each column')

```

6 P=[0 1 0;0 0 1;1 0 0];
7 disp(P, 'P=');
8 disp(P', 'inv(P)=P''=');
9 disp(P'*P, 'And,P''*P=');
10 //end

```

---

### Scilab code Exa 3.4.3 Projection onto a plane

```

1 //page 175
2 clear;
3 close;
4 clc;
5 disp('If we project b=(x,y,z) onto the x-y plane
      then its projection is p=(x,y,0),and is the sum
      of projection onto x- any y-axes.')
```

6  $b = \text{rand}(3,1)$ ;

7  $q1 = [1;0;0]$ ;

8  $\text{disp}(q1, 'q1=')$ ;

9  $q2 = [0;1;0]$ ;

10  $\text{disp}(q2, 'q2=')$ ;

11  $P = q1 * q1' + q2 * q2'$ ;

12  $\text{disp}(P, 'Overall projection matrix ,P=')$ ;

13  $\text{disp}('and, P[x;y;z]=[x;y;0]')$

14  $\text{disp}('Projection onto a plane=sum of projections
 onto orthonormal q1 and q2.')$

15 //end

---

### Scilab code Exa 3.4.4 Least squares fitting of data

```

1 //page 166
2 clear;

```



```

3  close;
4  clc;
5  disp('y=C+Dt');
6  disp('Ax=b');
7  A=[1 -3;1 0;1 3];
8  disp(A, 'A=');
9  y=rand      (3,1);
10 disp(y, 'y=');
11 disp('the columns of A are orthogonal,so')
12 x=zeros(1,2);
13 disp(([1 1 1]*y)/(A(:,1) '*A(:,1)), 'C^ =');
14 disp([-3 0 3]*y)/(A(:,2) '*A(:,2)), 'D^ =')
15 disp('C^ gives the besy fit ny horizontal line ,
        whereas D^t is the best fit by a straight line
        through the origin.')
16 //end

```

---

### Scilab code Exa 3.4.5 Gram Schmidt process

```

1  //page 166
2  clear;
3  close;
4  clc;
5  A=[1 0 1;1 0 0;2 1 0];//independent vectors stored
   in columns of A
6  disp(A, 'A=');
7  [m,n]=size(A);
8  for k=1:n
9      V(:,k)=A(:,k);
10     for j=1:k-1
11         R(j,k)=V(:,j) '*A(:,k);
12         V(:,k)=V(:,k)-R(j,k)*V(:,j);
13     end
14     R(k,k)=norm(V(:,k));
15     V(:,k)=V(:,k)/R(k,k);

```

```
16 end
17 disp(V, 'Q=')
```

---

# Chapter 4

## Determinants

Scilab code Exa 4.3.1 Determinant of a matrix is the product of its pivots

```
1 clear;
2 close;
3 clc;
4 n=input('Enter the value of n:');
5 for i=1
6     for j=i;
7         a(i,j)=2;
8         a(i,j+1)=-1;
9     end
10 end
11 for i=2:n-1
12     for j=i
13         a(i,j-1)=-1;
14         a(i,j)=2;
15         a(i,j+1)=-1;
16     end
17 end
18 for i=n
19     for j=i
20         a(i,j-1)=-1;
21         a(i,j)=2;
```

```

22     end
23 end
24 disp(a, 'a=');
25 [L,D,U]=lu(a)
26 determinant=1;
27 for i=1:n
28     determinant=determinant*D(i,i);
29 end
30 disp(determinant, 'Determinant=')
31 //end

```

---

**Scilab code Exa 4.3.2** Calculation of determinant of a matrix by using cofactors

```

1 clear;
2 close;
3 clc;
4 disp('For a 3*3 matrix:');
5 disp('det A=a11(a22a33-a23a32)+a12(a23a31-a21a33)+
      a13(a21a32-a22a31)');
6 //end

```

---

**Scilab code Exa 4.3.3** Calculation of determinant of a matrix by using cofactors

```

1 //page 214
2 clear;
3 close;
4 clc;
5 A=[2 -1 0 0;-1 2 -1 0;0 -1 2 -1;0 0 -1 2];
6 disp(A, 'A=');
7 [m,n]=size(A)
8 a=A(1,:);

```

```

9 c=[];
10 for l=1:4
11     B=A([1:0,2:4],[1:l-1,l+1:4]);
12     c1l=(-1)^(1+l)*det(B);
13     c=[c;c1l];
14 end
15 d=a*c;
16 disp(d)

```

---

**Scilab code Exa 4.4.1** Inverse of a sum matrix is a difference matrix

```

1 //282
2 clear;
3 close;
4 clc;
5 A=[1 1 1;0 1 1;0 0 1];
6 disp(A,'A=');
7 n=size(A,1); d=1:n-1;
8 B=zeros(n); AA=[A,A;A,A]';
9 for j=1:n
10     for k=1:n
11         B(j,k)=det(AA(j+d,k+d));
12     end
13 end
14 disp(B,'Adjoint of A:');
15 disp(B/det(A),'inv(A):');
16 //end

```

---

**Scilab code Exa 4.4.2** Cramers rule

```

1 //page 222
2 clear;
3 close;

```

```
4  clc;  
5  //x1+3x2=0  
6  //2x1+4x2=6  
7  A=[1 3;2 4];  
8  b=[0;6];  
9  X1=[0 3;6 4];  
10 X2=[1 0;2 6];  
11 disp(det(X1)/det(A), 'x1=');  
12 disp(det(X2)/det(A), 'x2=');  
13 //end
```

---

# Chapter 5

## Eigenvalues and Eigenvectors

Scilab code Exa 5.1.1 Eigenvalues and eigenvectors

```
1 //page 238
2 clear;
3 close;
4 clc;
5 A=[3 0;0 2];
6 eig=spec(A);
7 [V,Val]=spec(A);
8 disp(eig,'Eigen values:');
9 x1=V(:,1);
10 x2=V(:,2);
11 disp(x1,x2,'Eigen vectors:');
12 //end
```

---

Scilab code Exa 5.1.2 Eigenvalues and eigenvectors

```
1 //page 238
2 clear;
3 close;
```

```

4 clc;
5 disp('The eigen values of a projection matrix are 1
      or 0. ');
6 P=[1/2 1/2;1/2 1/2];
7 eig=spec(P);
8 [V,Val]=spec(P);
9 disp(eig, 'Eigen values: ');
10 x1=V(:,1);
11 x2=V(:,2);
12 disp(x1,x2, 'Eigen vectors: ');
13 //end

```

---

#### Scilab code Exa 5.2.1 Diagonalization

```

1 //page 238
2 clear;
3 close;
4 clc;
5 A=[1/2 1/2;1/2 1/2];
6 [V,Val]=spec(A);
7 disp(Val, 'Eigenvalue matrix: ');
8 disp(V, 'S=');
9 disp(A*V, 'AS=S*eigenvaluematrix ');
10 disp('Therefore  $\text{inv}(S)*A*S=\text{eigenvalue matrix}$  ');
11 //end

```

---

#### Scilab code Exa 5.2.2 Diagonalization

```

1 //page 238
2 clear;
3 close;
4 clc;

```



```

5 disp('The eigenvalues themselves are not so clear
      for a rotation.')
```

```

6 disp('90 degree rotation')
```

```

7 K=[0 -1;1 0];
```

```

8 disp(K, 'K=')
```

```

9 eig=spec(K);
```

```

10 [V,Val]=spec(K);
```

```

11 disp(eig, 'Eigen values:')
```

```

12 x1=V(:,1);
```

```

13 x2=V(:,2);
```

```

14 disp(x1,x2, 'Eigen vectors:');
```

```

15 //end
```

---

### Scilab code Exa 5.2.3 Powers and Products

```

1 //page 249
```

```

2 clear;
```

```

3 close;
```

```

4 clc;
```

```

5 disp('K is rotation through 90 degree ,then K^2 is
      rotation through 180 degree and inv(k is rotation
      through -90 degree)')
```

```

6 K=[0 -1;1 0];
```

```

7 disp(K, 'K=')
```

```

8 disp(K*K, 'K^2=')
```

```

9 disp(K*K*K, 'K^3=')
```

```

10 disp(K^4, 'K^4=')
```

```

11 [V,D]=spec(K);
```

```

12 disp('K^4 is a complete rotation through 360 degree.
      ')
13 disp(D, 'Eigen value matrix ,D of K: ');
```

```

14 disp(D^4, 'and also D^4=')
```

```

15 //end
```

---

### Scilab code Exa 5.3.1 Difference equations

```
1 //page 249
2 clear;
3 close;
4 clc;
5 A=[0 4;0 1/2];
6 disp(A, 'A=');
7 eig=spec(A);
8 disp(eig, 'Eigen values:');
9 [v,D]=spec(A);
10 u0=[v(:,1)]; //Taking u0 as the 1st eigen Vector.
11 for k=0:5
12     disp(k, 'k=');
13     u=A*u0;
14     disp(u, 'U(k+1)(K from 0 to 5)')
15     u0=u;
16 end
17 u0=[v(:,2)]; //Taking u0 as the 2nd eigen vector.
18 for k=0:5
19     disp(k, 'k=');
20     u=A*u0;
21     disp(u, 'U(k+1)=')
22     u0=u;
23 end
```

---

### Scilab code Exa 5.5.1 Complex matrices

```
1 //page282
2 clear;
3 close;
4 clc;
```

```
5 i=sqrt(-1);
6 x=3+4*i;
7 disp(x, 'x=');
8 x_=conj(x);
9 disp(x*x_, 'xx_=');
10 r=sqrt(x*x_);
11 disp(r, 'r=')
12 //end
```

---

Scilab code Exa 5.5.2 Inner product of a complex matrix

```
1 //282
2 clear;
3 close;
4 clc;
5 i=sqrt(-1);
6 x=[1 i]';
7 y=[2+1*i 2-4*i]';
8 disp(x'*x, 'Length of x squared:');
9 disp(y'*y, 'Length of y squared:');
10 //end
```

---

# Chapter 6

## Positive Definite Matrices

Scilab code Exa 6.1.1 Definite versus indefinite

```
1 //313
2 clear;
3 close;
4 clc;
5 disp('f(x,y)=x^2-10*x*y+y^2');
6 a=1;
7 c=1;
8 deff('[f]=f(x,y)', 'f=x^2-10*x*y+y^2');
9 disp(f(1,1), 'f(1,1)=');
10 disp('The conditions a>0 and c>0 ensure that f(x,y)
      is positive on the x and y axes. But this
      function is negative on the line x=y, because b
      =-10 overwhelms a and c. ');
11 //end
```

---

Scilab code Exa 6.1.3 Maxima Minima And Saddle points

```
1 //315
```

```

2 clear;
3 close;
4 clc;
5 disp('f(x,y)=2*x^2+4*x*y+y^2');
6 A=[2 2;2 1];
7 a=1;
8 c=1;
9 b=2;
10 disp(a*c,'ac=');
11 disp(b^2,'b^2=');
12 disp('Saddle point ,as ac<b^2');

```

---

**Scilab code Exa 6.1.4** Maxima Minima And Saddle points

```

1 //315
2 clear;
3 close;
4 clc;
5 disp('f(x,y)=2*x^2+4*x*y+y^2');
6 A=[2 2;2 1];
7 a=0;
8 c=0;
9 b=1;
10 disp(a*c,'ac=');
11 disp(b^2,'b^2=');
12 disp('Saddle point ,as ac<b^2');

```

---

**Scilab code Exa 6.2.2** Maxima Minima And Saddle points

```

1 //313
2 clear;
3 close;
4 clc;

```

```

5 disp('f(x,y)=x^2+4*x*y+y^2');
6 a=1;
7 c=1;
8 deff(' [ f]=f(x,y)', 'f=x^2+4*x*y+y^2');
9 disp(f(0,0), 'f(0,0)=')
10 disp('Here 2b=4 it still does not ensure a minimum
        ,the sign of b is of no importance. Neither F nor
        f has a minimum at(0,0) because f(1,-1)=-1. ')
11 //end

```

---

### Scilab code Exa 6.3.1 Singular value decomposition

```

1 //332
2 clear;
3 close;
4 clc;
5 A=[-1 2 2]';
6 disp(A, 'A=');
7 [U diagnol V]=svd(A);
8 disp(U, 'U=');
9 disp(diagnol, 'diagnol=');
10 disp(V', 'V''=');
11 disp(U*diagnol*V', 'A=U*diagnol*V''')
12 //end

```

---

### Scilab code Exa 6.3.2 Singular value decomposition

```

1 //332
2 clear;
3 close;
4 clc;
5 A=[-1 1 0;0 -1 1];
6 disp(A, 'A=');

```

```

7 [U diagnl V]=svd(A);
8 disp(U, 'U=');
9 disp(diagnl, 'Diagonal=');
10 disp(V, 'V''=');
11 disp(U*diagnl*V, 'A=U*diagonal*V''=')
12 //end

```

---

### Scilab code Exa 6.3.3 Polar decomposition

```

1 //332
2 clear;
3 close;
4 clc;
5 A=[1 -2;3 -1];
6 disp(A, 'A=');
7 [U S V]=svd(A);
8 Q=U*V';
9 S=V*S*V';
10 disp(Q, 'Q=');
11 disp(S, 'S=');
12 disp(Q*S, 'A=SQ=')
13 //end

```

---

### Scilab code Exa 6.3.4 Reverse polar decomposition

```

1 //332
2 clear;
3 close;
4 clc;
5 A=[1 -2;3 -1];
6 disp(A, 'A=');
7 [U diag1 V]=svd(A);
8 Q=U*V';

```

```
9 S=[2 1;1 3];
10 disp(Q, 'Q=');
11 disp(S, 'S=')
12 disp(S'*Q, 'A=S''Q=')
13 //end
```

---



# Chapter 7

## Computations with Matrices

Scilab code Exa 7.4.1 Jacobi Method

```
1 //page 238
2 clear;
3 close;
4 clc;
5 A=[2 -1;-1 2];
6 S=[2 0;0 2];
7 T=[0 1;1 0];
8 p=inv(S)*T;
9 b=[2 2]';
10 x=zeros(2,1);
11 disp(x, 'intial v & w:')
12 x_1=zeros(1,2);
13 for k=0:25
14     x_1=p*x+inv(S)*b;
15     x=x_1;
16     disp(k, 'k=')
17     disp(x_1, 'v(k+1) & w(k+1)=');
18 end
```

---

### Scilab code Exa 7.4.2 Gauss Seidel method

```
1 //page 238
2 clear;
3 close;
4 clc;
5 A=[2 -1;-1 2];
6 S=[2 0;-1 2];
7 T=[0 1;0 0];
8 b=rand(2,1);
9 p=inv(S)*T;
10 x=zeros(2,1);
11 disp(x, 'intial v & w: ')
12 x_1=zeros(1,2);
13 for k=0:25
14     x_1=p*x+inv(S)*b;
15     x=x_1;
16     disp(k, 'k=')
17     disp(x_1, 'v(k+1) & w(k+1)=');
18 end
```

---

# Chapter 8

## Linear Programming and Game Theory

Scilab code Exa 8.2.2 Minimize  $cx$  subject to  $x$  greater than or equal to zero and  $Ax$  equals to  $b$

```
1 //page 238
2 clear;
3 close;
4 clc;
5 A=[1 0 1 6 2;0 1 1 0 3];
6 b=[8 9]';
7 c=[0 0 7 -1 -3]';
8 lb=[0 0 0 0 0]';
9 ub=[];
10 [x,lagr,f]=linpro(c,A,b,lb,ub);
11 disp(x,'New corner:');
12 disp(f,'Minimum cost:');
13 //end
```

---