Scilab Textbook Companion for Linear Algebra by K. Hoffman and R. Kunze[1]

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Book Description

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Scilab numbering policy used in this document and the relation to the above book.

Exa Example (Solved example)

Eqn Equation (Particular equation of the above book)

AP Appendix to Example(Scilab Code that is an Appednix to a particular Example of the above book)

For example, Exa 3.51 means solved example 3.51 of this book. Sec 2.3 means a scilab code whose theory is explained in Section 2.3 of the book.

Contents

List of Scilab Codes

Chapter 1

Linear Equations

Scilab code Exa 1.5 Elementary Row Operations

```
//page 8
//Example 1.5
clear;
close;
clc;
a = [2 -1 3 2; 1 4 0 -1; 2 6 -1 5];
disp(a,'a=');
disp('Applying row transformations:');
disp('R1 = R1-2*R2');
a(1,:) = a(1,:) - 2*a(2,:);
disp(a,'a = ');
disp('R3 = R3-2*R2');
a(3,:) = a(3,:) - 2*a(2,:);
disp(a,'a = ');
disp('R3 = R3/-2');
a(3,:) = -1/2*a(3,:);
disp(a,'a = ');
disp('R2 = R2-4*R3');
a(2,:) = a(2,:) - 4*a(3,:);
disp(a,'a = ');
disp('R1 = R1+9*R3');
```

```
a(1,:) = a(1,:) + 9*a(3,:);
disp(a,'a = ');
disp('R1 = R1*2/15');
a(1,:) = a(1,:) * 2/15;
disp(a,'a = ');
disp('R2 = R2+2*R1');
a(2,:) = a(2,:) + 2*a(1,:);
disp(a,'a = ');
disp('R3 = R3-R1/2');
a(3,:) = a(3,:) - 1/2*a(1,:);
disp(a,'a = ');
disp('We get the system of equations as:');
disp('2*x1 - x2 + 3*x3 + 2*x4 = 0');
disp('x1 + 4*x2 - x4 = 0');
disp('2*x1 + 6* x2 - x3 + 5*x4 = 0');
disp('and');
disp('x2 - 5/3*x4 = 0','x1 + 17/3*x4 = 0','x3 -
   11/3*x4 = 0');
disp('now by assigning any rational value c to x4 in
    system second, the solution is evaluated as:');
disp('(-17/3*c,5/3,11/3*c,c)');
//end
```

Scilab code Exa 1.6 Elementary Row Operations

```
//page 9
//Example 1.6
clear;
close;
clc;
a=[-1 %i;-%i 3;1 2];
disp(a,'a = ');
disp('Applying row transformations:');
disp('R1 = R1+R3 and R2 = R2 + i *R3');
a(1,:) = a(1,:) +a(3,:);
```

```
a(2,:) = a(2,:) + %i * a(3,:);
disp(a,'a = ');
disp('R1 = R1 * (1/2+i)');
a(1,:) = 1/(2 + %i) * a(1,:);
disp(a,'a = ');
disp('R2 = R2-R1*(3+2i) and R3 = R3 - 2 *R1');
a(2,:) = round(a(2,:) - (3 + 2 * %i) * a(1,:));
a(3,:) = round(a(3,:) - 2 * a(1,:));
disp(a,'a = ');
disp('Thus the system of equations is:');
disp('x1 + 2*x2 = 0','-i*x1 + 3*x2 = 0','-x1+i*x2 = 0');
disp('It has only trivial solution x1 = x2 = 0');
//end
```

Scilab code Exa 1.7 Row reduced echelon Matrix

```
//page 9
//Example 1.7
clear;
close;
clc;
n = rand();
n = round(n*10);
disp(eye(n,n));
printf('This is an Identity matrix of order %d * %d',n,n);
disp('And It is a row reduced matrix.');
//end
```

Scilab code Exa 1.8 Row reduced echelon Matrix

```
//page 12
```

```
//Example 1.8
clear;
close;
clc;
n = rand();
n = round(n*10);
disp(eye(n,n));
printf('This is an Identity matrix of order %d * %d',n,n);
disp('And It is a row reduced matrix.');
m = rand();
n = rand();
m = round(m*10);
n = round(n*10);
disp(zeros(m,n));
printf('This is an Zero matrix of order %d * %d',m,n);
disp('And It is also a row reduced matrix.');
a = [0 1 -3 0 1/2;0 0 0 1 2;0 0 0 0 0];
disp(a,'a = ');
disp('This is a non-trivial row reduced matrix.');
//end
```

Scilab code Exa 1.9 System of Equations

```
//page 14
//Example 1.9
clear;
close;
clc;
A = [1 -2 1;2 1 1;0 5 -1];
disp(A,'A = ');
disp('Applying row transformations:');
disp('R2 = R2 - 2*R1');
A(2,:) = A(2,:) - 2*A(1,:);
```

```
disp(A,'A = ');
disp('R3 = R3 - R2');
A(3,:) = A(3,:) - A(2,:);
disp(A,'A = ');
disp('R2 = 1/5*R2');
A(2,:) = 1/5*A(2,:);
disp(A,'A = ');
disp('R1 = R1 - 2*R2');
A(1,:) = A(1,:) + 2*A(2,:);
disp(A,'A = ');
disp('The condition that the system have a solution
   is:');
disp('2*y1 - y2 + y3 = 0');
disp('where, y1,y2,y3 are some scalars');
disp('If the condition is satisfied then solutions
   are obtained by assigning a value c to x3');
disp('Solutions are:');
disp('x2 = 1/5*c + 1/5*(y2 - 2*y1)','x1 = -3/5*c +
   1/5*(y1 + 2*y2)');
//end
```

Scilab code Exa 1.10 Product of Matrices

```
//page 17
//Example 1.10
clear;
close;
clc;
//Part a
a = [1 0;-3 1];
b = [5 -1 2;15 4 8];
disp(a,'a=');
disp(b,'b=');
disp(a*b,'ab = ');
disp('
```

```
------------------------------------------------------------------------
');
//Part b
a = [1 0;-2 3;5 4;0 1];
b = [0 6 1;3 8 -2];
disp(a,'a=');
disp(b,'b=');
disp(a*b,'ab = ');
disp('------------------------------------------------------------------------');
//Part c
a = [2 1;5 4];
b = [1;6];
disp(a,'a=');
disp(b,'b=');
disp(a*b,'ab = ');
disp('------------------------------------------------------------------------');
//Part d
a = [-1;3];
b = [2 4];
disp(a,'a=');
disp(b,'b=');
disp(a*b,'ab = ');
disp('------------------------------------------------------------------------');
//Part e
a = [2 4];
b = [-1;3];
disp(a,'a=');
disp(b,'b=');
disp(a*b,'ab = ');
disp('------------------------------------------------------------------------');
```

```
//Part f
a = [0 1 0;0 0 0;0 0 0];
b = [1 -5 2;2 3 4;9 -1 3];
disp(a,'a=');
disp(b,'b=');
disp(a*b,'ab = ');
disp('
    ______________________________________________________________________
    ');
//Part g
a = [1 -5 2;2 3 4;9 -1 3];
b = [0 1 0;0 0 0;0 0 0];
disp(a,'a=');
disp(b,'b=');
disp(a*b,'ab = ');
//end
```

Scilab code Exa 1.14 Inverse of a matrix

```
//page 22
//Example 1.14
clear;
close;
clc;
a = [0 1;1 0];
disp(a,'a = ');
disp(inv(a),'inverse a = ');
//end
```

Scilab code Exa 1.15 Inverse of a matrix

```
//page 25
//Example 1.15
```

```
clear;
close;
clc;
a = [2 -1;1 3];
disp(a,'a = ');
b = a;                  //Temporary variable to store a
disp('Applying row tranformations');
disp('Interchange R1 and R2');
x = a(1,:);
a(1,:) = a(2,:);
a(2,:) = x;
disp(a,'a = ');
disp('R2 = R2 - 2 * R1');
a(2,:) = a(2,:) - 2 * a(1,:);
disp(a,'a = ');
disp('R2 = R2 *1/(-7)');
a(2,:) = (-1/7) * a(2,:);
disp(a,'a = ');
disp('R1 = R1 - 3 * R2');
a(1,:) = a(1,:) - 3 * a(2,:);
disp(a,'a = ');
disp('Since a  has become an identity matrix. So, a
   is invertible');
disp('inverse of a = ');
disp(inv(b)); //a was stored in b
//end
```

Scilab code Exa 1.16 Inverse of a matrix

```
//page 25
//Example 1.16
clear;
close;
clc;
a = [1 1/2 1/3;1/2 1/3 1/4;1/3 1/4 1/5];
```

```
disp(a,'a = ');
b = eye(3,3);
disp(b,'b = ');
disp('Applying row transformations on a and b simultaneously,');
disp('R2 = R2 - 1/2 * R1 and R3 = R3 - 1/3*R1');
a(2,:) = a(2,:) - 1/2 * a(1,:);
a(3,:) = a(3,:) - 1/3 * a(1,:);
b(2,:) = b(2,:) - 1/2 * b(1,:);
b(3,:) = b(3,:) - 1/3 * b(1,:);
disp(a,'a = ');
disp(b,'b = ');
disp('R3 = R3 - R2');
a(3,:) = a(3,:) -  a(2,:);
b(3,:) = b(3,:) -  b(2,:);
disp(a,'a = ');
disp(b,'b = ');
disp('R2 = R2 * 12 and R3 = R3 * 180');
a(2,:) = a(2,:) *12;
a(3,:) = a(3,:) * 180;
b(2,:) = b(2,:) * 12;
b(3,:) = b(3,:) * 180;
disp(a,'a = ');
disp(b,'b = ');
disp('R2 = R2 - R3 and R1 = R1 - 1/3*R3');
a(2,:) = a(2,:) - a(3,:);
a(1,:) = a(1,:) - 1/3 * a(3,:);
b(2,:) = b(2,:) - b(3,:);
b(1,:) = b(1,:) - 1/3 * b(3,:);
disp(a,'a = ');
disp(b,'b = ');
disp('R1 = R1 - 1/2 * R2');
a(1,:) = a(1,:) -  1/2 * a(2,:);
b(1,:) = b(1,:) -  1/2 * b(2,:);
disp(round(a),'a = ');
disp(b,'b = ');
disp('Since, a = identity matrix of order 3*3. So, b is inverse of a');
```

```
disp(b,'inverse(a) = ');
//end
```

Chapter 2

Vector Spaces

Scilab code Exa 2.8 Vector Subspace

```
//page 37
//Example 2.8
clear;
clc;
close;
a1 = [1 2 0 3 0];
a2 =[0 0 1 4 0];
a3 = [0 0 0 0 1];
disp(a1,'a1 = ');
disp(a2,'a2 = ');
disp(a3,'a3 = ');
disp('By theorem 3, vector a is in subspace W of F^5
    spanned by a1, a2, a3');
disp('if and only if there exist scalars c1, c2, c3
   such that');
disp('a= c1a1 + c2a2 + c3a3');
disp('So, a = (c1,2*c1,c2,3c1+4c2,c3)');
c1 = -3;
c2 = 1;
c3 = 2;
a = c1*a1 + c2*a2 + c3*a3;
```

```
disp(c1,'c1 = ');
disp(c2,'c2 = ');
disp(c3,'c3 = ');
disp(a,'Therefore, a = ');
disp('This shows, a is in W');
disp('And (2,4,6,7,8) is not in W as there is no
    value of c1 c2 c3 that satisfies the equation');
//end
```

Scilab code Exa 2.10 Row space of matrix

```
//page 38
//Example 2.10
clear;
clc;
close;
A = [1 2 0 3 0;0 0 1 4 0;0 0 0 0 1];
disp(A,'A = ');
disp('The subspace of F^5 spanned by a1 a2 a3(row
    vectors of A) is called row space of A.');
a1 = A(1,:);
a2 = A(2,:);
a3 = A(3,:);
disp(a1,'a1 = ');
disp(a2,'a2 = ');
disp(a3,'a3 = ');
disp('And, it is also the row space of B.');
B = [1 2 0 3 0;0 0 1 4 0;0 0 0 0 1;-4 -8 1 -8 0];
disp(B,'B = ');
//end
```

Scilab code Exa 2.11 Space of polynomial function

```
//page 39
//Example 2.11
clear;
clc;
close;
disp('V is the space of all polynomial functions
   over F.');
disp('S contains the functions as:')
x = poly(0,"x");
n = round(rand()*10);
disp(n,'n = ');
for i = 0 : n
    f = x^i;
    printf('f%d(x) = ',i);
    disp(f);
end
disp('Then, V is the subspace spanned by set S.');
//end
```

Scilab code Exa 2.12 Linear Dependency

```
//page 41
//Example 2.12
clear;
clc;
close;
a1 = [3 0 -3];
a2 = [-1 1 2];
a3 = [4 2 -2];
a4 = [2 1 1];
disp(a1, 'a1 = ');
disp(a2, 'a2 = ');
disp(a3, 'a3 = ');
disp(a4, 'a4 = ');
t = 2 * a1 + 2 * a2 - a3 + 0 * a4;
```

```
disp(' = 0',t,' Since, 2 * a1 + 2 * a2 - a3 + 0 * a4 = ');
disp('a1,a2,a3,a4 are linearly independent');
e1 = [1 0 0];
e2 = [0 1 0];
e3 = [0 0 1];
disp(e1, 'Now, e1 = ');
disp(e2, 'e2 = ');
disp(e3, 'e3 = ');
disp('Also, e1,e2,e3 are linearly independent.');
//end
```

Scilab code Exa 2.13 Standard basis of Matrix

```
//page 41
//Example 2.13
clear;
clc;
close;
disp('S is the subset of F^n consisting of n vectors.');
n = round(rand() *10 + 1);
disp(n,'n = ');
I = eye(n,n);
for i = 0 : n-1
    e = I(i+1,:);
    printf('e%d = ',i+1);
    disp(e);
end
disp('x1,x2,x3...xn are the scalars in F');
disp('Putting a = x1*e1 + x2*e2 + x3*e3 + .... + xn*en');
disp('So, a = (x1,x2,x3,...,xn)');
disp('Therefore, e1,e2..,en span F^n');
disp('a = 0 if x1 = x2 = x3 = .. = xn = 0');
```

```
disp('So,e1,e2,e3,..,en are linearly independent.');
disp('The set S = {e1,e2,..,en} is called standard basis of F^n');
//end
```

Scilab code Exa 2.20 Inverse of a matrix

```
//page 54
//Example 2.20
clear;
clc;
close;
P = [-1 4 5; 0 2 -3; 0 0 8];
disp(P,'P = ');
disp(inv(P),'inverse(P) = ');
a1 = P(:,1);
a2 = P(:,2);
a3 = P(:,3);
disp('The vectors forming basis of F^3 are a1'', a2'', a3''');
disp(a1','a1'' = ');
disp(a2','a2'' = ');
disp(a3','a3'' = ');
disp('The coordinates x1'',x2'',x3'' of vector a = [x1,x2,x3] is given by inverse(P)*[x1; x2; x3]');
t = -10*a1 - 1/2*a2 - a3;
disp(t,'And, -10*a1'' - 1/2*a2'' - a3'' = ');
//end
```

Scilab code Exa 2.21 Standard basis of matrix

```
//page 60
//Example 2.21
```

```
clear;
clc;
close;
a1 = [1 2 2 1];
a2 = [0 2 0 1];
a3 = [-2 0 -4 3];
disp('Given row vectors are:');
disp(a1,'a1 = ');
disp(a2,'a2 = ');
disp(a3,'a3 = ');
disp('The matrix A from these vectors will be:');
A = [a1; a2; a3];
disp(A,'A = ');
disp('Finding Row reduced echelon matrix of A that is given by R');
disp('And applying same operations on identity matrix Q such that R = QA');
Q = eye(3,3);
disp(Q,'Q = ');
T = A;                 //Temporary matrix to store A
disp('Applying row transformations on A and Q,we get ');
disp('R1 = R1-R2');
A(1,:) = A(1,:) - A(2,:);
Q(1,:) = Q(1,:) - Q(2,:);
disp(A,'A = ');
disp(Q,'Q = ');
disp('R3 = R3 + 2*R1');
A(3,:) = A(3,:) + 2*A(1,:);
Q(3,:) = Q(3,:) + 2*Q(1,:);
disp(A,'A = ');
disp(Q,'Q = ');
disp('R3 = R3/3');
A(3,:) = 1/3*A(3,:);
Q(3,:) = 1/3*Q(3,:);
disp(A,'A = ');
disp(Q,'Q = ');
disp('R2 = R2/2');
```

```
A(2,:) = 1/2*A(2,:);
Q(2,:) = 1/2*Q(2,:);
disp(A,'A = ');
disp(Q,'Q = ');
disp('R2 = R2 - 1/2*R3');
A(2,:) = A(2,:) - 1/2*A(3,:);
Q(2,:) = Q(2,:) - 1/2*Q(3,:);
disp(A,'A = ');
disp(Q,'Q = ');
R = A;
A = T;
disp('Row reduced echelon matrix:');
disp(R,'R = ');
disp(Q,'Q =');
//part a
disp(rank(R),'rank of R = ');
disp('Since, Rank of R is 3, so a1, a2, a3 are independent');
//part b
disp('Now, basis for W can be given by row vectors of R i.e. p1,p2,p3');
disp('b is any vector in W. b = [b1 b2 b3 b4]');
disp('Span of vectors p1,p2,p3 consist of vector b with b3 = 2*b1');
disp('So,b = b1p1 + b2p2 + b4p3');
disp('And,[p1 p2 p3] = R = Q*A');
disp('So, b = [b1 b2 b3]* Q * A');
disp('hence, b = x1a1 + x2a2 + x3a3 where x1 = [b1 b2 b4] * Q(1) and so on'); //Equation 1
//part c
disp('Now, given 3 vectors a1'' a2'' a3'':');
c1 = [1 0 2 0];
c2 = [0 2 0 1];
c3 = [0 0 0 3];
disp(c1,'a1'' = ');
disp(c2,'a2'' = ');
disp(c3,'a3'' = ');
disp('Since a1'' a2'' a3'' are all of the form (y1
```

```
    y2 y3 y4) with y3 = 2*y1, hence they are in W.');
disp('So, they are independent.');
//part d
c = [c1; c2; c3];
P = eye(3,3);
for i = 1:3
    b1 = c(i,1);
    b2 = c(i,2);
    b4 = c(i,4);
    x1 = [b1 b2 b4] * Q(:,1);
    x2 = [b1 b2 b4]*Q(:,2);
    x3 = [b1 b2 b4]*Q(:,3);
   P(:,i) = [x1; x2; x3];
end
disp('Required matrix P such that X = PX'' is:');
disp(P,'P = ');
//end
```

Scilab code Exa 2.22 Standard basis of Matrix

```
//page 63
//Example 2.22
clear;
clc;
close;
A = [1 2 0 3 0;1 2 -1 -1 0;0 0 1 4 0;2 4 1 10 1;0 0
    0 0 1];
disp(A,'A = ');
//part a
T = A;                    //Temporary storing A in T
disp('Taking an identity matrix P:');
P = eye(5,5);
disp(P,'P = ');
disp('Applying row transformations on P and A to get
    a row reduced echelon matrix R:');
```

```
disp('R2 = R2 - R1 and R4 = R4 - 2* R1');
A(2,:) = A(2,:) - A(1,:);
P(2,:) = P(2,:) - P(1,:);
A(4,:) = A(4,:) - 2 * A(1,:);
P(4,:) = P(4,:) - 2 * P(1,:);
disp(A,'A = ');
disp(P,'P = ');
disp('R2 = -R2 , R3 = R3 - R1 + R2 and R4  = R4 - R1 + R2');
A(2,:) = -A(2,:);
P(2,:) = -P(2,:);
A(3,:) = A(3,:) - A(2,:);
P(3,:) = P(3,:) - P(2,:);
A(4,:) = A(4,:) - A(2,:);
P(4,:) = P(4,:) - P(2,:);
disp(A,'A = ');
disp(P,'P = ');
disp('Mutually interchanging R3, R4 and R5');
x = A(3,:);
A(3,:) = A(5,:);
y = A(4,:);
A(4,:) = x;
A(5,:) = y - A(3,:);
x = P(3,:);
P(3,:) = P(5,:);
y = P(4,:);
P(4,:) = x;
P(5,:) = y - P(3,:);
R = A;
A = T;
disp(R,'Row reduced echelon matrix R = ');
disp(P,'Invertible Matrix P = ');
disp('Invertible matrix P is not unique. There can be many that depends on operations used to reduce A');
disp('---------------------------------------------------------');
//part b
disp('For the basis of row space W of A, we can take
```

```
the non-zero rows of R');
disp('It can be given by p1, p2, p3');
p1 = R(1,:);
p2 = R(2,:);
p3 = R(3,:);
disp(p1,'p1 = ');
disp(p2,'p2 = ');
disp(p3,'p3 = ');
disp('----------------------------------------------');
//part c
disp('The row space W consists of vectors of the form:');
disp('b = c1p1 + c2p2 + c3p3');
disp('i.e. b = (c1,2*c1,c2,3*c1+4*c2,c3) where, c1 c2 c3 are scalars.');
disp('So, if b2 = 2*b1 and b4 = 3*b1 + 4*b3  =>  (b1,b2,b3,b4,b5) = b1p1 + b3p2 + b5p3');
disp('then,(b1,b2,b3,b4,b5) is in W');
disp('----------------------------------------------');
//part d
disp('The coordinate matrix of the vector (b1,2*b1,b2,3*b1+4*b2,b3) in the basis (p1,p2,p3) is column matrix of b1,b2,b3 such that:');
disp('  b1');
disp('  b2');
disp('  b3');
disp('----------------------------------------------');
//part e
disp('Now, to write each vector in W as a linear combination of rows of A:');
disp('Let b = (b1,b2,b3,b4,b5) and if b is in W, then');
disp('we know,b = (b1,2*b1,b3,3*b1 + 4*b3,b5)  =>  [b1,b3,b5,0,0]*R');
disp('=> b = [b1,b3,b5,0,0] * P*A  =>  b = [b1+b3,-b3,0,0,b5] * A');
disp('if b = (-5,-10,1,-11,20)');
b1 = -5;
```

```
b2 = -10;
b3 = 1;
b4 = -11;
b5 = 20;
x = [b1 + b3,-b3,0,0,b5];
disp(']',A,'[','*',')',x,'(','b = ');
disp('-----------------------------------------------');
//part f
disp('The equations in system RX = 0 are given by R
    * [x1 x2 x3 x4 x5]');
disp('i.e., x1 + 2*x2 + 3*x4');
disp('x3 + 4*x4');
disp('x5');
disp('so, V consists of all columns of the form');
disp('[','X=');
disp('  -2*x2 - 3*x4');
disp('  x2');
disp('  -4*x4');
disp('  x4');
disp('  0');
disp('where x2 and x4  are arbitrary',']');
disp('-----------------------------------------------');
//part g
disp('Let x2 = 1,x4 = 0 then the given column forms
    a basis of V');
x2 = 1;
x4 = 0;
disp([-2*x2-3*x4; x2; -4*x4; x4; 0]);
disp('Similarly,if x2 = 0,x4 = 1 then the given
    column forms a basis of V');
x2 = 0;
x4 = 1;
disp([-2*x2-3*x4; x2; -4*x4; x4; 0]);
disp('-----------------------------------------------');
//part h
disp('The equation AX = Y has solutions X if and
    only if');
disp('-y1 + y2 + y3 = 0');
```

```
disp('-3*y1 + y2 + y4 -y5 = 0');
disp('where, Y = (y1 y2 y3 y4 y5)');
//end
```

Chapter 3

Linear Transformations

Scilab code Exa 3.6 Linear Transformation function

```
//page 70
//Example 3.6
clc;
clear;
close;
a1 = [1 2];
a2 = [3 4];
disp(a1,'a1 = ');
disp(a2,'a2 = ');
disp('a1 and a2 are linearly independent and hence
   form a basis for R^2');
disp('According to theorem 1, there is a linear
   transformation from R^2 to R^3 with the
   transformation functions as:');
Ta1 = [3 2 1];
Ta2 = [6 5 4];
disp(Ta1,'Ta1 = ');
disp(Ta2,'Ta2 = ');
disp('Now, we find scalars c1 and c2 for that we
   know T(c1a1 + c2a2) = c1(Ta1) + c2(Ta2))');
disp('if(1,0) = c1(1,2) + c2(3,4), then ');
```

```
c = inv([a1;a2]') * [1;0];
c1 = c(1,1);
c2 = c(2,1);
disp(c1,'c1 = ');
disp(c2,'c2 = ');
disp('The transformation function T(1,0) will be:');
T = c1*Ta1 + c2*Ta2;
disp(T,'T(1,0) = ');
//end
```

Scilab code Exa 3.12 Singular and onto linear transformation

```
//page 81
//Example 3.12
clc;
clear;
close;
x = round(rand(1,2) * 10);
x1 = x(1);
x2 = x(2);
T = [x1+x2 x1];
disp(x1,'x1 = ');
disp(x2,'x2 = ');
printf('T(%d,%d) = ',x1,x2);
disp(T);
disp('If, T(x1,x2) = 0, then');
disp('x1 = x2 = 0');
disp('So, T is non-singular');
disp('z1,z2 are two scalars in F');
z1 = round(rand() * 10);
z2 = round(rand() * 10);
disp(z1,'z1 = ');
disp(z2,'z2 = ');
x1 = z2;
x2 = z1 - z2;
```

```
disp(x1,'So, x1 = ');
disp(x2,'x2 = ');
disp('Hence, T is onto.');
Tinv = [z2 z1-z2];
disp(Tinv,'inverse(T) = ');
//end
```

Scilab code Exa 3.14 Standard Ordered Basis

```
//page 89
//Example 3.14
clc;
clear;
close;
disp('T is a linear operator on F^2 defined as:');
disp('T(x1,x2) = (x1,0)');
disp('B = {e1,e2} is a standard ordered basis for F
    ^2,then');
x1 = 1;
x2 = 0;
Te1 = [x1 0];
x1 = 0;
x2 = 1;
Te2 = [x1 0];
disp(Te1,'So, Te1 = T(1,0) = ');
disp(Te2,'So, Te2 = T(0,1) = ');
disp('so,matrix T in ordered basis B is: ');
T = [Te1; Te2];
disp(T,'T = ');
//end
```

Scilab code Exa 3.15 Matrix in Ordered basis

```
//page 89
//Example 3.15
clc;
clear;
close;
disp('Differentiation operator D is defined as:');
D = zeros(4,4);
x = poly(0,"x");
for i= 1:4
    t= i-1;
    f = derivat(x^t);
    printf('(Df%d)(x) = ',i);
    disp(f);
    if ~(i == 1) then
    D(i-1,i) = i-1;
    end
end
disp('Matrix of D in ordered basis is:');
disp(D,'[D] = ');
//end
```

Scilab code Exa 3.16 Standard Ordered Basis

```
//page 92
//Example 3.16
clc;
clear;
close;
disp('T is a linear operator on R^2 defined as T(x1,
   x2) = (x1,0)');
disp('So, the matrix T in standard ordered basis B =
    {e1,e2} is ');
T = [1 0 ;0 0];
disp(T,'[T]B = ');
disp('Let B'' is the ordered basis for R^2
```

```
    consisting of vectors:');
E1 = [1 1];
E2 = [2 1];
disp(E1,'E1 = ');
disp(E2,'E2 = ');
P = [E1;E2]'
disp(P,'So, matrix P = ');
Pinv = inv(P);
disp(Pinv,'P inverse = ');
T1 = Pinv*T*P;
disp(T1,'So, matrix T in ordered basis B'' is [T]B''
    = ');
//end
```

Scilab code Exa 3.17 Matrix in ordered basis

```
//page 93
//Example 3.17
clc;
clear;
close;
t = poly(0,"t");
disp('g1 = f1');
disp('g2 = t*f1 + f2');
disp('g3 = t^2*f1 + 2*t*f2 + f3');
disp('g4 = t^3*f1 + 3*t^2*f2 + 3*t*f3 + f4');
P = [1 t t^2 t^3;0 1 2*t 3*t^2;0 0 1 3*t;0 0 0 1];
disp(P,'P = ');
disp(inv(P),'inverse P = ');
disp('Matrix of differentiation operator D in
    ordered basis B is:'); //As found in example 15
D = [0 1 0 0;0 0 2 0;0 0 0 3;0 0 0 0];
disp(D,'D = ');
disp('Matrix of D in ordered basis B'' is:');
disp(inv(P)*D*P,'inverse(P) * D * P = ');
```

```
//end
```

Scilab code Exa 3.19 Trace of a matrix

```
//page 98
//Example 3.19
clc;
clear;
close;
function [tr] = trace_matrix(M,n)
    for i = 1 : n
    tr = tr + M(i,i);
    end
endfunction
n = round(rand() * 10 + 2);
disp(n,'n = ');
A = round(rand(n,n) * 10);
disp(A,'A = ');
tr = 0;
disp('Trace of A:');
tr1 = trace_matrix(A,n);
disp(tr1,'tr(A) = ');
disp('----------------------------------------');
c = round(rand() * 10 + 2);
disp(c,'c = ');
B = round(rand(n,n) * 10);
disp(B,'B = ');
disp('Trace of B:');
tr2 = trace_matrix(B,n);
disp(tr2,'tr(B) = ');
disp(c*tr1+tr2,'tr(cA + B) = ');
//end
```

Scilab code Exa 3.23 Linear functional on vector space

```
//page 103
//Example 3.23
clc;
clear;
close;
disp('Matrix represented by given linear functionals
    on R^4:');
A = [1 2 2 1;0 2 0 1;-2 0 -4 3];
disp(A,'A = ');
T = A;                  //Temporary matrix to store A
disp('To find Row reduced echelon matrix of A given
   by R:')
disp('Applying row transformations on A,we get');
disp('R1 = R1-R2');
A(1,:) = A(1,:) - A(2,:);
disp(A,'A = ');
disp('R3 = R3 + 2*R1');
A(3,:) = A(3,:) + 2*A(1,:);
disp(A,'A = ');
disp('R3 = R3/3');
A(3,:) = 1/3*A(3,:);
disp(A,'A = ');
disp('R2 = R2/2');
A(2,:) = 1/2*A(2,:);
disp(A,'A = ');
disp('R2 = R2 - 1/2*R3');
A(2,:) = A(2,:) - 1/2*A(3,:);
disp(A,'A = ');
R = A;
A = T;
disp('Row reduced echelon matrix of A is:');
disp(R,'R = ');
disp('Therefore,linear functionals g1,g2,g3 span the
    same subspace of (R^4)* as f1,f2,f3 are given by
   :');
disp('g1(x1,x2,x3,x4) = x1 + 2*x3');
```

```
disp('g1(x1,x2,x3,x4) = x2');
disp('g1(x1,x2,x3,x4) = x4');
disp('The subspace consists of the vectors with');
disp('x1 = -2*x3');
disp('x2 = x4 = 0');
//end
```

Scilab code Exa 3.24 Linear functional on vector space

```
//page 104
//Example 3.24
clc;
clear;
close;
disp('W be the subspace of R^5 spanned by vectors:');
a1 = [2 -2 3 4 -1];
a2 = [-1 1 2 5 2];
a3 = [0 0 -1 -2 3];
a4 = [1 -1 2 3 0];
disp(a1,'a1 = ');
disp(a2,'a2 = ');
disp(a3,'a3 = ');
disp(a4,'a4 = ');
disp('Matrix A by the row vectors a1,a2,a3,a4 will be:');
A = [a1;a2;a3;a4];
disp(A,'A = ');
disp('After Applying row transformations, we get the row reduced echelon matrix R of A;');
T = A;                    //Temporary matrix to store A
//R1 = R1 - R4 and R2 = R2 + R4
A(1,:) = A(1,:) - A(4,:);
A(2,:) = A(2,:) + A(4,:);
```

```
//R2 = R2/2
A(2,:) = 1/2 * A(2,:);
//R3 = R3 + R2 and R4 = R4 - R1
A(3,:) = A(3,:) + A(2,:);
A(4,:) = A(4,:) - A(1,:);
//R3 = R3 - R4
A(3,:) = A(3,:) - A(4,:);
//R3 = R3/3
A(3,:) = 1/3 * A(3,:);
//R2 = R2 - R3
A(2,:) = A(2,:) - A(3,:);
//R2 = R2/2 and R4 = R4 - R2 - R3
A(2,:) = 1/2 * A(2,:);
A(4,:) = A(4,:) - A(2,:) - A(3,:);
//R1 = R1 - R2 + R3
A(1,:) = A(1,:) - A(2,:) + A(3,:);
R = A;
A = T;
disp(R,'R = ');
disp('Then we obtain all the linear functionals f by assigning arbitrary values to c2 and c4');
disp('Let c2 = a, c4 = b then c1 = a+b, c3 = -2b, c5 = 0.');
disp('So, W0 consists all linear functionals f of the form');
disp('f(x1,x2,x3,x4,x5) = (a+b)x1 + ax2 -2bx3 + bx4');
disp('Dimension of W0 = 2 and basis {f1,f2} can be found by first taking a = 1, b = 0. Then a = 0,b = 1');
//end
```

Chapter 4

Polynomials

Scilab code Exa 4.3 Algebra of linear operators

```
//page 121
//Example 4.3
clc;
clear;
close;
disp('C is the field of complex numbers');
x = poly(0,"x");
f = x^2 + 2;
disp(f,'f = ');
//part a
disp('if a = C and z belongs to C, then f(z) = z^2 + 2');
disp(horner(f,2),'f(2) = ');
disp(horner(f,(1+%i)/(1-%i)),'f(1+%i/1-%i) = ');
disp('----------------------------------------');
//part b
disp('If a is the algebra of all 2*2 matrices over C and');
B = [1 0;-1 2];
disp(B,'B = ');
disp(2*eye(2,2) + B^2,'then, f(B) = ');
```

```
disp('—————————————————————————————————————————');
//part c
disp('If a is algebra of all linear operators on C^3
   ');
disp('And T is element of a as:');
disp('T(c1,c2,c3) = (i*2^1/2*c1,c2,i*2^1/2*c3)');
disp('Then, f(T)(c1,c2,c3) = (0,3*c2,0)');
disp('—————————————————————————————————————————');
//part d
disp('If a is the algebra of all polynomials over C'
   );
g = x^4 + 3*%i;
disp(g,'And, g = ');
disp(horner(f,g),'Then f(g) = ');
//end
```

Scilab code Exa 4.7 Ideal of a polynomial

```
//page 131
//Example 4.7
clc;
clear;
close;
x = poly(0,"x");
p1 = x + 2;
p2 = x^2 + 8*x + 16;
disp('M = (x+2)F[x] + (x^2 + 8x + 16)F[x]');
disp('We assert , M = F[x]');
disp('M contains:');
t = p2 - x*p1;
disp(t);
disp('And hence M contains:');
disp(t - 6*p1);
disp('Thus the scalar polynomial 1 belongs to M as
   well all its multiples.')
```

```
//end
```

Scilab code Exa 4.8 G C D of polynomials

```
//page 133
//Example 4.8
clc;
clear;
close;
x = poly(0,"x");
//part a
p1 = x + 2;
p2 = x^2 + 8*x + 16;
disp(p1,'p1 = ');
disp(p2,'p2 = ');
disp('M = (x+2)F[x] + (x^2 + 8x + 16)F[x]');
disp('We assert , M = F[x]');
disp('M contains:');
t = p2 - x*p1;
disp(t);
disp('And hence M contains:');
disp(t - 6*p1);
disp('Thus the scalar polynomial 1 belongs to M as
   well all its multiples');
disp('So, gcd(p1,p2) = 1');
disp('------------------------------------------------
   ');
//part b
p1 = (x - 2)^2*(x+%i);
p2 = (x-2)*(x^2 + 1);
disp(p1,'p1 = ');
disp(p2,'p2 = ');
disp('M = (x - 2)^2*(x+%i)F[x] + (x-2)*(x^2 + 1');
disp('The ideal M contains p1 - p2 i.e.,');
disp(p1 - p2);
```

```
disp('Hence it contains (x-2)(x+i), which is monic and divides both,');
disp('So, gcd(p1,p2) = (x-2)(x+i)');
disp('----------------------------------------------------');
//end
```

Scilab code Exa 4.9 Ideal of a Polynomial

```
//page 133
//Example 4.9
clc;
clear;
close;
disp('M is the ideal in F[x] generated by:');
disp('(x-1)*(x+2)^2');
disp('(x+2)^2*(x+3)');
disp('(x-3)','and');
x = poly(0,"x");
p1 = (x-1)*(x+2)^2;
p2 = (x+2)^2*(x-3);
p3 = (x-3);
disp('M = (x-1)*(x+2)^2 F[x] + (x+2)^2*(x-3) + (x-3)');
disp('Then M contains:');
t = 1/2*(x+2)^2*((x-1) - (x-3));
disp(t);
disp('i.e., M contains (x+2)^2');
disp('and since, (x+2)^2 = (x-3)(x-7) - 17');
disp('So M contains the scalar polynomial 1.');
disp('So, M = F[x] and given polynomials are relatively prime.');
//end
```

Scilab code Exa 4.10 Reducible Polynomial

```
//page 135
//Example 4.10
clc;
clear;
close;
x = poly(0,"x");
P = x^2 + 1;
disp(P,'P = ');
disp('P is reducible over complex numbers as: ');
disp('=',P);
disp('(x-i)(x+i)');
disp('Whereas, P is irreducible over real numbers as
    :.');
disp('=',P);
disp('(ax + b)(a''x + b'')');
disp('For, a,a'',b,b'' to be in R,');
disp('aa'' = 1');
disp('ab'' + ba'' = 0');
disp('bb'' = 1');
disp('=> a^2 + b^2 = 0');
disp('=> a = b = 0');
//end
```

Chapter 5

Determinants

Scilab code Exa 5.3 Two linear function

```
//page 143
//Example 5.3
clc;
clear;
close;
A = round(rand(2,2) *10 );
disp(A,'A = ');
D1 = A(1,1)*A(2,2);
D2 = - A(1,2)*A(2,1);
disp(D1,'D1(A) = ');
disp(D2,'D2(A) = ');
disp(D1 + D2,'D(A) = D1(A) + D2(A) = ');
disp('That is, D is a 2-linear function.');
//end
```

Scilab code Exa 5.4 Alternating 3 Linear Functions

```
//page 145
```

```
//Example 5.4
clc;
clear;
close;
x = poly(0,"x");
A = [x 0 -x^2;0 1 0;1 0 x^3];
disp(A,'A = ');
disp('e1,e2,e3 are the rows of 3*3 identity matrix, then');
T = eye(3,3);
e1 = T(1,:);
e2 = T(2,:);
e3 = T(3,:);
disp(e1,'e1 = ');
disp(e2,'e2 = ');
disp(e3,'e3 = ');
disp('D(A) = D(x*e1 - x^2*e3, e2, e1 + x^3*e3)');
disp('Since, D is linear as a function of each row,');
disp('D(A) = x*D(e1,e2,e1 + x^3*e3) - x^2*D(e3,e2,e1 + x^3*e3)');
disp('D(A) = x*D(e1,e2,e1) + x^4*D(e1,e2,e3) - x^2*D(e3,e2,e1) - x^5*D(e3,e2,e3)');
disp('As D is alternating, So');
disp('D(A) = (x^4 + x^2)*D(e1,e2,e3)');
//end
```

Scilab code Exa 5.5 Determinant of a matrix

```
//page 147
//Example 5.5
clc;
clear;
close;
function [E1 , E2 , E3] = determinant(A)
```

```
    E1 = A(1,1)*det([A(2,2) A(2,3);A(3,2) A(3,3)]) - A(2,1)*det([A(1,2) A(1,3);A(3,2) A(3,3)]) + A(3,1)*det([A(1,2) A(1,3);A(2,2) A(2,3)]);
    E2 = -A(1,2)*det([A(2,1) A(2,3);A(3,1) A(3,3)]) + A(2,2)*det([A(1,1) A(1,3);A(3,1) A(3,3)]) + A(3,2)*det([A(1,1) A(1,3);A(2,1) A(2,3)]);
    E3 = A(1,3)*det([A(2,1) A(2,2);A(3,1) A(3,2)]) - A(2,3)*det([A(1,1) A(1,2);A(3,1) A(3,2)]) + A(3,3)*det([A(1,1) A(1,2);A(2,1) A(2,2)]);
endfunction

//part a
x = poly(0,"x");
A = [x-1 x^2 x^3;0 x-2 1;0 0 x-3];
disp(A,'A = ');
[E1, E2, E3] = determinant(A);
disp(E1,'E1(A) = ');
disp(E2,'E2(A) = ');
disp(E3,'E3(A) = ');
disp('————————————————————————————————————————');
//part b
A = [0 1 0;0 0 1;1 0 0];
disp(A,'A = ');
[E1, E2, E3] = determinant(A);
disp(E1,'E1(A) = ');
disp(E2,'E2(A) = ');
disp(E3,'E3(A) = ');
//end
```

Scilab code Exa 5.6 Determinant of a matrix

```
//page 158
//Example 5.6
clc;
clear;
```

```
close;
disp('Given Matrix:');
A = [1 -1 2 3; 2 2 0 2; 4 1 -1 -1;1 2 3 0];
disp(A,'A = ');
disp('After, Subtracting muliples of row 1 from rows 2 3 4');
disp('R2 = R2 - 2*R1');
A(2,:) = A(2,:) - 2 * A(1,:);
disp('R3 = R3 - 4*R1');
A(3,:) = A(3,:) - 4 * A(1,:);
disp('R4 = R4 - R1');
A(4,:) = A(4,:) - A(1,:);
disp(A,'A = ');
T = A;                        //Temporary matrix to store A
disp('We obtain the same determinant as before.');
disp('Now, applying some more row transformations as :');
disp('R3 = R3 - 5/4 * R2');
T(3,:) = T(3,:) - 5/4 * T(2,:);
disp('R4 = R4 - 3/4 * R2');
T(4,:) = T(4,:) - 3/4 * T(2,:);
B = T;
disp('We get B as:');
disp(B,'B = ');
disp('Now,determinant of A and B will be same');
disp(det(B),'det A = det B = ');
//end
```

Scilab code Exa 5.7 Inverse of a matrix

```
//page 160
//Example 5.7
clc;
clear;
```

```
close;
x = poly(0,"x");
A = [x^2+x x+1;x-1 1];
B = [x^2-1 x+2;x^2-2*x+3 x];
disp(A,'A = ');
disp(B,'B = ');
disp(det(A),'det A = ');
disp(det(B),'det B = ');
disp('Thus, A is not invertible over K whereas B is
    invertible');
disp(inv(A)*det(A),'adj A = ');
disp(inv(B)*det(B),'adj B = ');
disp('(adj A)A = (x+1)I');
disp('(adj B)B = -6I');
disp(inv(B),'B inverse = ');
//end
```

Scilab code Exa 5.8 Inverse of a matrix

```
//page 161
//Example 5.8
clc;
clear;
close;
A = [1 2;3 4];
disp(A,'A = ');
d = det(A);
disp(d,'det A = ','Determinant of A is:');
ad = (det(A) * eye(2,2)) / A;
disp(ad,'adj A = ','Adjoint of A is:');
disp('Thus, A is not invertible as a matrix over the
     ring of integers.');
disp('But, A can be regarded as a matrix over field
    of rational numbers.');
in = inv(A);
```

```
//The A inverse matrix given in book has a wrong
    entry of 1/2. It should be -1/2.
disp(in,'inv(A) = ','Then, A is invertible and
    Inverse of A is:');
//end
```

Chapter 6

Elementary Canonical Forms

Scilab code Exa 6.1 Characteristic Polynomial of a matrix

```
//page 184
//Example 6.1
clc;
clear;
close;
disp('Standard ordered matrix for Linear operator T
   on R^2 is:');
A = [0 -1;1 0];
disp(A,'A = ');
disp('The characteristic polynomial for T or A is:')
x = poly(0,"x");
p = detr(x*eye(2,2)-A);
disp(p);
disp('Since this polynomial has no real roots,T has
   no characteristic values.');
//end
```

Scilab code Exa 6.2 Characteristic Polynomial of a matrix

```
//page 184
//Example 6.2
clc;
clear;
close;
A = [3 1 -1; 2 2 -1;2 2 0];
disp(A,'A = ');
disp('Characteristic polynomial for A is:');
p = poly(A,"x");
disp(p);
disp('or');
disp('(x-1)(x-2)^2');
r = roots(p);
[m,n] = size(A);
disp('The characteristic values of A are:');
disp(round(r));
B = A-eye(m,n);
disp(B,'Now, A-I = ');
disp(rank(B),'rank of A - I= ');
disp('So, nullity of T-I = 1');
a1 = [1 0 2];
disp(a1,'The vector that spans the null space of T-I = ');
B = A-2*eye(m,n);
disp(B,'Now,A-2I = ');
disp(rank(B),'rank of A - 2I= ');
disp('T*alpha = 2*alpha if alpha is a scalar multiple of a2');
a2 = [1 1 2];
disp(a2,'a2 = ');
//end
```

Scilab code Exa 6.3 Characteristic Polynomial of a matrix

```
//page 187
```

```
//Example 6.3
clc;
clear;
close;
disp('Standard ordered matrix for Linear operator T on R^3 is:');
A = [5 -6 -6; -1 4 2; 3 -6 -4];
disp(A,'A = ');
disp('xI - A = ');
B = eye(3,3);
x = poly(0,"x");
P = x*B - A;
disp(P);
disp('Applying row and column transformations:');
disp('C2 = C2 - C3');
P(:,2) = P(:,2) - P(:,3);
disp('=>');
disp(P);
disp('Taking (x-2) common from C2');
c = x-2;
P(:,2) = P(:,2) / (x-2);
disp('=>');
disp(' * ', c);
disp(P);
disp('R3 = R3 + R2');
P(3,:) = P(3,:) + P(2,:);
disp('=>');
disp(' * ', c);
disp(P);
P = [P(1,1) P(1,3); P(3,1) P(3,3)];
disp('=>');
disp(' * ', c);
disp(P);
disp('=>');
disp(' * ',c);
disp(det(P));
disp('This is the characteristic polynomial');
disp(A-B,'Now, A - I = ');
```

```
disp(A-2*B,'And, A- 2I = ');
disp(rank(A-B),'rank(A-I) = ');
disp(rank(A-2*B),'rank(A-2I) = ');
disp('W1,W2 be the spaces of characteristic vectors
   associated with values 1,2');
disp('So by theorem 2, T is diagonalizable');
a1 = [3 -1 3];
a2 = [2 1 0];
a3 = [2 0 1];
disp(a1,'Null space of (T- I) i.e basis of W1 is
   spanned by a1 = ');
disp('Null space of (T- 2I) i.e. basis of W2 is
   spanned by vectors x1,x2,x3 such that x1 = 2x1 +
   2x3');
disp('One example are;');
disp(a2,'a2 = ');
disp(a3,'a3 = ');
disp('The diagonal matrix is:');
D = [1 0 0 ;0 2 0;0 0 2];
disp(D,'D = ');
disp('The standard basis matrix is denoted as:');
P = [a1;a2;a3]';
disp(P,'P = ');
disp(A*P,'AP = ');
disp(P*D,'PD = ');
disp('That is, AP = PD');
disp('=>  inverse(P)*A*P = D');
//end
```

Scilab code Exa 6.4 Diagonalizable Operator

```
//page 193
//Example 6.4
clc;
clear;
```

```
close;
x = poly(0,"x");
A = [5 -6 -6; -1 4 2; 3 -6 -4];      //Matrix given in Example 3
disp(A,'A = ');
f = (x-1)*(x-2)^2;
disp('Characteristic polynomial of A is:');
disp('f = (x-1)(x-2)^2');
disp(f,'i.e., f = ');
p = (x-1)*(x-2);
disp((A-eye(3,3))*(A-2 * eye(3,3)),'(A-I)(A-2I) = ');
disp('Since, (A-I)(A-2I) = 0. So, Minimal polynomial for above is:');
disp(p,'p = ');
disp('----------------------------------------------');
A = [3 1 -1; 2 2 -1;2 2 0];      //Matrix given in Example 2
disp(A,'A = ');
f = (x-1)*(x-2)^2;
disp('Characteristic polynomial of A is:');
disp('f = (x-1)(x-2)^2');
disp(f,'i.e., f = ');
disp((A-eye(3,3))*(A-2 * eye(3,3)),'(A-I)(A-2I) = ');
disp('Since, (A-I)(A-2I) is not equal to 0. T is not diagonalizable. So, Minimal polynomial cannot be p.');
disp('----------------------------------------------');
A = [0 -1;1 0];
disp(A,'A = ');
f = x^2 + 1;
disp('Characteristic polynomial of A is:');
disp(f,'f = ');
disp(A^2 + eye(2,2),'A^2 + I = ');
disp('Since, A^2 + I = 0, so minimal polynomial is');
p = x^2 + 1;
```

```
disp(p,'p = ');
//end
```

Scilab code Exa 6.5 Characteristic Polynomial of matrix

```
//page 197
//Example 6.5
clc;
clear;
close;
A = [0 1 0 1;1 0 1 0;0 1 0 1;1 0 1 0];
disp(A,'A = ');
disp('Computing powers on A:');
disp(A*A,'A^2 = ');
disp(A*A*A,'A^3 = ');
deff('[p] = p(x)','p = x^3 - 4*x');
disp('if p = x^3 - 4x, then');
disp(p(A),'p(A) = ');
x = poly(0,"x");
f = x^3 - 4*x;
disp(f,'Minimal polynomial for A is: ');
disp(roots(f),'Characteristic values for A are:');
disp(rank(A),'Rank(A) = ');
disp(round(poly(A,"x")),'So, from theorem 2,
    characteristic polynomial for A is:');
//end
```

Scilab code Exa 6.12 Symmetric and skew symmetric matrix

```
//page 210
//Example 6.12
clc;
clear;
```

```
close;
A = round(rand(3,3) * 10);
disp(A,'A = ');
disp('A transpose is:');
disp(A','A'' = ');
if A' == A then
    disp('Since, A'' = A, A is a symmetric matrix.');
else
    disp('Since, A'' is not equal to A, A is not a symmetric matrix.');
end
if A' == -A then
    disp('Since, A'' = -A, A is a skew-symmetric matrix.');
else
    disp('Since, A'' is not equal to -A, A is not a skew-symmetric matrix.');
end
A1 = 1/2*(A + A');
A2 = 1/2*(A - A');
disp('A can be expressed as sum of A1 and A2');
disp('i.e., A = A1 + A2');
disp(A1,'A1 = ');
disp(A2,'A2 = ');
disp(A1 + A2,'A1 + A2 = ');
//end
```

Chapter 7

The Rational and Jordan Forms

Scilab code Exa 7.3 Linear operator annihilator

```
//page 239
//Example 7.3
clc;
clear;
close;
A = [5 -6 -6;-1 4 2;3 -6 -4];
disp(A,'A = ');
f = poly(A,"x");
disp('Characteristic polynomial for linear operator
   T on R^3 will be:');
disp(f,'f = ');
disp('or');
disp('(x-1)(x-2)^2');
x = poly(0,"x");
disp('The minimal polynomial for T is:');
p = (x-1)*(x-2);
disp(p,'p = ');
disp('or');
disp('p = (x-1)(x-2)');
disp('So in cyclic decomposition of T, a1 will have
   p as its T-annihilator.');
```

```
disp('Another vector a2 that generate cyclic
    subspace of dimension 1 will have its T-
    annihilator as p2.');
p2 = x-2;
disp(p2,'p2 = ');
disp(p*p2,'pp2 = ');
disp('i.e., pp2 = f');
disp('Therefore, A is similar to B');
B = [0 -2 0;1 3 0;0 0 2];
disp(B,'B = ');
disp('Thus, we can see thet Matrix of T in ordered
    basis is B');
//end
```

Scilab code Exa 7.6 Characteristic and minimal polynomial of matrix

```
//page 247
//Example 7.6
clc;
clear;
close;
disp('A = ');
disp('2   0   0');
disp('a   2   0');
disp('b   c   -1');
a = 1;
b = 0;
c = 0;
A = [2 0 0;a 2 0;b c -1];
disp(A,'A = ');
disp('Characteristic polynomial for A is:');
disp(poly(A,"x"),'p = ');
disp('In this case, minimal polynomial is same as
    characteristic polynomial.');
disp('--------------------------------------------');
```

```
a = 0;
b = 0;
c = 0;
A = [2 0 0;a 2 0;b c -1];
disp(A,'A = ');
disp('Characteristic polynomial for A is:');
disp(poly(A,"x"),'p = ');
disp('In this case, minimal polynomial is:');
disp('(x-2)(x+1)');
disp('or');
x = poly(0,"x");
s = (x-2)*(x+1);
disp(s);
disp('(A-2I)(A+I) = ');
disp('0     0    0');
disp('3a    0    0');
disp('ac    0    0');
disp('if a = 0, A is similar to diagonal matrix.')
//end
```

Scilab code Exa 7.7 Characteristic and minimal polynomial of matrix

```
//page 247
//Example 7.7
clc;
clear;
close;
disp('A = ');
disp('2    0    0    0');
disp('1    2    0    0');
disp('0    0    2    0');
disp('0    0    a    2');
disp('Considering a = 1');
A = [2 0 0 0;1 2 0 0;0 0 2 0;0 0 1 2];
p = poly(A,"x");
```

```
disp('Characteristic polynomial for A is:');
disp(p,'p = ');
disp('or');
disp('(x-2)^4');
disp('Minimal polynomial for A =');
disp('(x-2)^2');
disp('For a = 0 and a = 1, characteristic and
   minimal polynomial are same.');
disp('But for a=0, the solution space of (A - 2I)
   has 3 dimension whereas for a = 1, it has 2
   dimension. ')
//end
```

Chapter 8

Inner Product Spaces

Scilab code Exa 8.1 Standard Inner Product

```
//page 271
//Example 8.1
clc;
clear;
close;
n = round(rand() * 10 + 2);
a = round(rand(1,n) * 10)
b = round(rand(1,n) * 10)
disp(n,'n = ');
disp(a,'a = ');
disp(b,'b = ');
disp(a*b','Then, (a|b) = ');
//end
```

Scilab code Exa 8.2 Standard Inner Product

```
//page 271
//Example 8.2
```

```
clc;
clear;
close;
a = round(rand(1,2) * 10)
b = round(rand(1,2) * 10)
disp(a,'a = ');
disp(b,'b = ');
x1 = a(1);
x2 = a(2);
y1 = b(1);
y2 = b(2);
t = x1*y1 - x2*y1 - x1*y2 + 4*x2*y2;
disp(t,'Then, a|b = ');
//end
```

Scilab code Exa 8.9 Standard Inner Product

```
//page 278
//Example 8.9
clc;
clear;
close;
a = round(rand(1,2) * 10);
x = a(1);
y = a(2);
b = [-y x];
disp(a,'(x,y) = ');
disp(b,'(-y,x) = ');
disp('Inner product of these vectors is:');
t = -x*y + y*x;
disp(t,'(x,y)|(-y,x) = ');
disp('So, these are orthogonal.');
disp('------------------------------------------------');
disp('If inner product is defined as:');
disp('(x1,x2)|(y1,y2) = x1y1- x2y1 - x1y2 + 4x2y2');
```

```
disp('Then, (x,y)|(-y,x) = -x*y+y^2-x^2+4*x*y = 0 if
    ,');
disp('y = 1/2(-3 + sqrt(13))*x');
disp('or');
disp('y = 1/2(-3 - sqrt(13))*x');
disp('Hence,');
if y == 1/2*(-3 + sqrt(13))*x | y == 1/2*(-3 - sqrt
    (13))*x then
disp(a);
disp('is orthogonal to');
disp(b);
else
disp(a);
disp('is not orthogonal to');
disp(b);
end
//end
```

Scilab code Exa 8.12 Orthogonal Vectors

```
//page 282
//Example 8.12
clc;
clear;
close;
b1 = [3 0 4];
b2 = [-1 0 7];
b3 = [2 9 11];
disp(b1,'b1 = ');
disp(b2,'b2 = ');
disp(b3,'b3 = ');
disp('Applying the Gram-Schmidt process to b1,b2,b3:
    ');
a1 = b1;
disp(a1,'a1 = ');
```

```
a2 = b2-((b2*b1')'/25*b1);
disp(a2,'a2 = ');
a3 = b3-((b3*b1')'/25*b1) - ((b3*a2')'/25*a2);
disp(a3,'a3 = ');
disp('{a1,a2,a3} are mutually orthogonal and hence forms orthogonal basis for R^3');
disp('Any arbitrary vector {x1,x2,x3} in R^3 can be expressed as:');
disp('y = {x1,x2,x3} = (3*x1 + 4*x3)/25*a1 + (-4*x1 + 3*x3)/25*a2 + x2/9*a3');
x1 = 1;
x2 = 2;
x3 = 3;
y = (3*x1 + 4*x3)/25*a1 + (-4*x1 + 3*x3)/25*a2 + x2/9*a3;
disp(x1,'x1 = ');
disp(x2,'x2 = ');
disp(x3,'x3 = ');
disp(y,'y = ');
disp('i.e. y = [x1 x2 x3], according to above equation.');
disp('Hence, we get the orthonormal basis as:');
disp(',',1/5*a1);
disp(',',1/5*a2);
disp(1/9*a3);
//end
```

Scilab code Exa 8.13 Orthogonal Vectors

```
//page 283
//Example 8.13
clc;
clear;
close;
A = rand(2,2);
```

```
A(1,:) = A(1,:) + 1; //so b1 is not equal to zero
a = A(1,1);
b = A(1,2);
c = A(2,1);
d = A(2,2);
b1 = A(1,:);
b2 = A(2,:);
disp(A,'A = ');
disp(b1,'b1 = ');
disp(b2,'b2 = ');
disp('Applying the orthogonalization process to b1,
   b2:');
a1 = [a b];
a2 = (det(A)/(a^2 + b^2))*[-b' a'];
disp(a1,'a1 = ');
disp(a2,'a2 = ');
disp('a2 is not equal to zero if and only if b1 and
   b2 are linearly independent.');
disp('That is, if determinant of A is non-zero.');
//end
```

Scilab code Exa 8.14 Orthogonal Projection

```
//page 286
//Example 8.14
clc;
clear;
close;
v = [-10 2 8];
u = [3 12 -1]
disp(v,'v = ');
disp(u,'v = ');
disp('Orthogonal projection of v1 on subspace W
   spanned by v2 is given by:');
a = ((u*v')')/(u(1)^2 + u(2)^2 + u(3)^2) * u;
```

```
disp(a);
disp('Orthogonal projection of R^3 on W is the linear transformation E given by:');
printf('(x1,x2,x3) -> (3*x1 + 12*x2 - x3)/%d * (3 12 -1)',(u(1)^2 + u(2)^2 + u(3)^2));
disp('Rank(E) = 1');
disp('Nullity(E)  = 2');
//end
```

Scilab code Exa 8.15 Orthogonal sets

```
//page 288
//Example 8.15
clc;
clear;
close;
//part c
disp('f = (sqrt(2)*cos(2*pi*t) + sqrt(2)*sin(4*pi*t))^2');
disp('Integration (f dt) in limits 0 to 1 = ');
x0 = 0;
x1 = 1;
X = integrate('(sqrt(2)*cos(2*%pi*t) + sqrt(2)*sin(4*%pi*t))^2','t',x0,x1);
disp(X);
//end
```

Scilab code Exa 8.17 Inner product space and orthogonal projection

```
//page 294
//Example 8.17
//Equation given in example 14 is used.
clc;
```

```
clear;
close;
function [m] = transform(x,y,z)
    x1 = 3*x;
    x2 = 12*y;
    x3 = -z;
    m = [x1 x2 x3];
endfunction

disp('Matrix of projection E in orthonormal basis is :');
t1 = transform(3,3,3);
t2 = transform(12,12,12);
t3 = transform(-1,-1,-1);
A = [t1; t2; t3];
disp(A,'A = 1/154 * ');
A1 = (conj(A))';
disp(A1,'A* = ');
disp('Since, E = E* and A = A*, then A is also the matrix of E*');
a1 = [154 0 0];
a2 = [145 -36 3];
a3 = [-36 10 12];
disp(a1,'a1 = ');
disp(a2,'a2 = ');
disp(a3,'a3 = ');
disp('{a1,a2,a3} is the basis.');
Ea1 = [9 36 -3];
Ea2 = [0 0 0];
Ea3 = [0 0 0];
disp(Ea1,'Ea1 = ');
disp(Ea2,'Ea2 = ');
disp(Ea3,'Ea3 = ');
B = [-1 0 0;-1 0 0;0 0 0];
disp('Matrix B of E in the basis is:');
disp(B,'B = ');
B1 = (conj(B))';
disp(B1,'B* = ');
```

```
disp('Since, B is not equal to B*, B is not the
    matrix of E*');
//end
```

Scilab code Exa 8.28 Unitary matrix

```
//page 307
//Example 8.28
clc;
clear;
close;
disp('x1 and x2  are two real nos. i.e., x1^2 + x2^2
    = 1');
x1 = rand();
x2 = sqrt(1 - x1^2);
disp(x1,'x1 = ');
disp(x2,'x2 = ');
B = [x1 x2 0;0 1 0;0 0 1];
disp(B,'B = ');
disp('Applying Gram-Schmidt process to B:')
a1 = [x1 x2 0];
a2 = [0 1 0] - x2 * [x1 x2 0];
a3 = [0 0 1];
disp(a1,'a1 = ');
disp(a2,'a2 = ');
disp(a3,'a3 = ');
U = [a1;a2/x1;a3];
disp(U,'U = ');
M = [1 0 0;-x2/x1 1/x1 0;0 0 1];
disp(M,'M = ')
disp(inv(M) * U,'inverse(M) * U = ');
disp('So, B = inverse(M) * U');
//end
```

Chapter 10

Bilinear Forms

Scilab code Exa 10.4 Bilinear Form of vectors

```
//page 363
//Example 10.4
clc;
clear;
close;
disp('a = [x1 x2]');
disp('b = [y1 y2]');
disp('f(a,b) = x1*y1 + x1*y2 + x2*y1 + x2*y2');
disp('so, f(a,b) = ');
disp('[x1  x2]  *  |1   1|  *   |y1|');
disp('              |1   1|      |y2|');
disp('So the matrix of f in standard order basis B = {e1,e2} is:');
fb = [1 1;1 1];
disp(fb,'[f]B = ');
P = [1 1;-1 1];
disp(P,'P = ');
disp('Thus, [f]B'' = P''*[f]B*P');
fb1 = P' * fb * P;
disp(fb1,'[f]B'' = ');
//end
```

Scilab code Exa 10.5 Bilinear Form of vectors

```
//page 365
//Example 10.5
clc;
clear;
close;
n = round(rand() * 10 + 2);
a = round(rand(1,n) * 10);
b = round(rand(1,n) * 10);
disp(n,'n = ');
disp(a,'a = ');
disp(b,'b = ');
f = a * b';
disp(f,'f(a,b) = ');
disp('f is non-degenerate billinear form on R^n.');
//end
```