Solving Differential Equation with Graphical Data Visualization using Python

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Introduction: why Differential Equation?

1. Differential equations are powerful tools for modeling data.
2. Differential equation sets the relationship between one or more derivatives and the function itself. For eg. $\frac{d^2y}{dx^2} = -w^2x$.
3. To understand how rapidly a system respond rather than its level of response.
4. Useful to understand the behaviour of the system over short and medium time period.
5. Differential equations are important when there are events at different time scale.
6. To get better estimate of functional parameters and there derivatives.

Why Python?

1. Open source OOP language.
2. Portable
   - Unix/Linux, Windows ,etc.
3. Interfacing to other languages, hence data acquisition is possible.
4. Easy to Learn, Read and Use.
5. Supports different Libraries and hence simple graphical visualisation.
6. Python is used with its extension modules NumPy, SciPy and Matplotlib, which adds more power to this scientific computing language to display the graphical results.
7. In the present paper we have solved various types of differential equations and computed the results in graphical format. The technique found useful and arouse the interest among the students at large.

General form of differential equation

The general form of first order and first degree differential equation

$$\frac{dy}{dx} = f(x, y)$$

Its Solution can be obtained by classifying them as follows
(i)Variable separable form,
(ii)Homogeneous and non homogeneous,
(iii)Exact differential equation,
(iv)Non Exact differential equation (Reducible to exact)
(v)Linear differential equation
(vi)Non linear differential equation (Reducible to linear).
We will discuss some of the applications in electric circuits and to correlates various parameters. The technique to simulate ODE systems using python is discussed. Simulation of such equation generates graph that gives values of function at any point.

Electric Circuit

To find current (or charge) through a circuit we can form a differential equation by using Kirchoff's Law.

R-L circuit

If resistance R ohms and inductance L henries are in series with *emf* of $e(t)$ volts, then by kirchoff's law differential equation is

$$\frac{di}{dt} + \frac{R}{L} i = \frac{e(t)}{L}$$

which linear whose integrating factor is $IF = e^{\frac{Rt}{L}}$ and solution

$$i.(IF) = \int \frac{e(t)}{L}.(IF) dt + c$$

where c is constant finding by using initial condition $i = 0$ at $t = 0$.

Exapmle

A generator having *emf* $100V$ is connected in series with 10Ω resistor and an inductor of $2H$. To find current i in circuit consider D.E.

$$\frac{di}{dt} + \frac{R}{L} i = \frac{e(t)}{L}$$

$IF = e^{\frac{Rt}{L}}$ and solution is

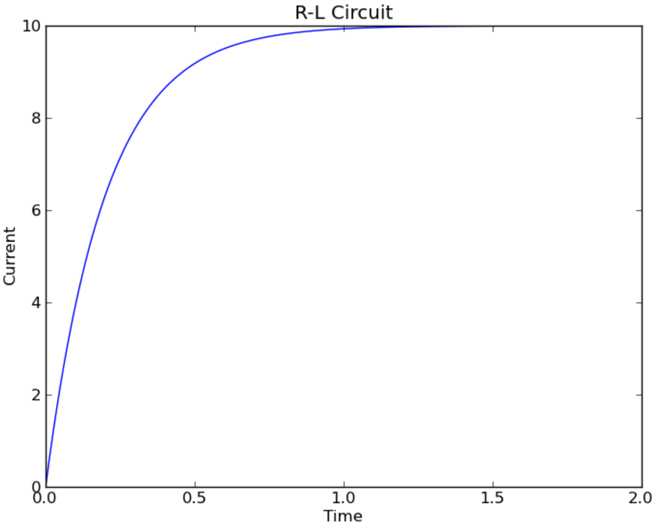
$$i = 10(1 - e^{-5t})$$

The solution for above equation can be obtained with python program as follows.

Programme

```
from pylab import *
t=arange(0,2,0.0001)
i=10*(1-exp(-5*t))
plot(t,i)
xlabel('Time')
ylabel('Current')
title('R-L Circuit')
show()
```

Graph



R-C circuit

If resistance R ohms and capacitance C Farads are in series with *emf* of $e(t)$ volts, then by kirchoff's law differential equation is

$$\frac{dq}{dt} + \frac{1}{RC}q = \frac{e(t)}{R}$$

which linear whose integrating factor is $IF = e^{\frac{Rt}{L}}$ and solution

$$q.(IF) = \int \frac{e(t)}{R}.(IF)dt + A$$

where A is constant finding by using initial condition $q = 0$ at $t = 0$. Also we can find current i through the circuit.

Example

A capacitor 0.1 Farad in series with a resistor 20Ω is charged from a battery $10V$. First we find charge q through circuit, consider differential equation

$$\frac{dq}{dt} + \frac{1}{RC} q = \frac{e(t)}{R}$$

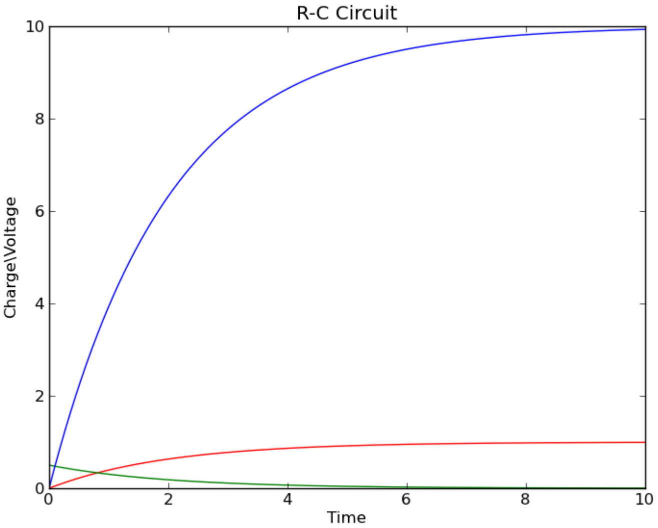
$IF = e^{\frac{1}{RC}t} = e^{0.5t}$ and charge q at any time t is $q = 1 - e^{-0.5t}$. So the current through circuit at any time t is $i = 0.5e^{-0.5t}$, also voltage $V = \frac{q}{C} = 10(1 - e^{-0.5t})$

The solution for above equation can be obtained with python program as follows.

Programme

```
from pylab import *
t=arange(0,10,0.01)
q=1-exp(-0.5*t)
i=0.5*exp(-0.5*t)
v=10*(1-exp(-0.5*t))
plot(t,q,'r')
plot(t,i, 'g')
plot(t,v,'b')
xlabel('Time')
ylabel(' Charge  Voltage')
title('R-C Circuit')
show()
```

Graph



Law of Natural Growth

Law of Natural Growth

If rate of change of a quantity y at any instant t is proportional to the quantity present at that time. The Differential equation of **growth is**

$$\frac{dy}{dt} = ky,$$

solution $x = Ae^{kt}$, where A is constant.
decay is

$$\frac{dy}{dt} = -ky,$$

solution $y = Ae^{-kt}$, where A is constant.

Example

In a culture of yeast, at each instant, the time rate of change of active ferment is proportional to the amount present. If the active ferment doubles in two hours, find amount at any time.
Let y be quantity of active ferment at any time t. The equation of fermentation yeast is

$$\frac{dy}{dt} = ky,$$

where k is constant and solution is

$$logy = kt + A.$$

Using initial condition $y = y_0$ at $t = 0$ so $A = logy_0$. Hence $log(\frac{y}{y_0}) = kt$. The active ferment double in two hours i.e. $y = 2y_0$ at $t = 2$ $k = \frac{1}{2}log2$. Therefore

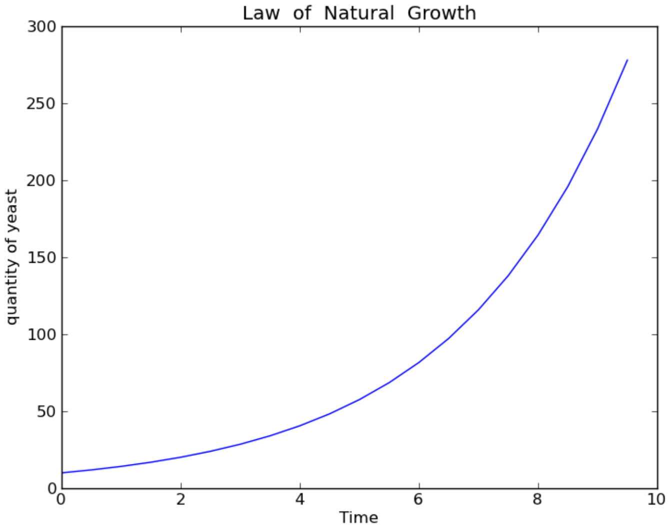
$$y = y_0 e^{\frac{t}{2}log2}$$

Programme

Programme

$$t = arange(0, 10, .5), y0 = 10$$
$$y = 10 * e(0.35 * t)$$
$$plot(t, y)$$
$$xlabel('Time')$$
$$ylabel('quantityofyeast')$$
$$title('Law\ of\ Natural\ Growth')$$

Graph


Law of Natural Growth
quantity of yeast
Time
0
50
100
150
200
250
300
0
2
4
6
8
10

Newton's Law of cooling

Newton's Law of cooling

Let θ be a initial temperature of a body and θ_0 be a surrounding temperature, then by Newton's law of cooling

$$\frac{d\theta}{dt} = -k(\theta - \theta_0),$$

where k is constant and solution is

$$\theta = \theta_0 + Ae^{kt}$$

where A is constant.

Example

Example

A body originally at $80^{\circ}c$ cools down to $60^{\circ}c$ in 20 minutes, the temperature of the air being $40^{\circ}c$. Then the temperature of the body at ant time t is

$$\theta = 80 + e^{-\frac{log2}{20}t}$$

Example

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Programme

Example

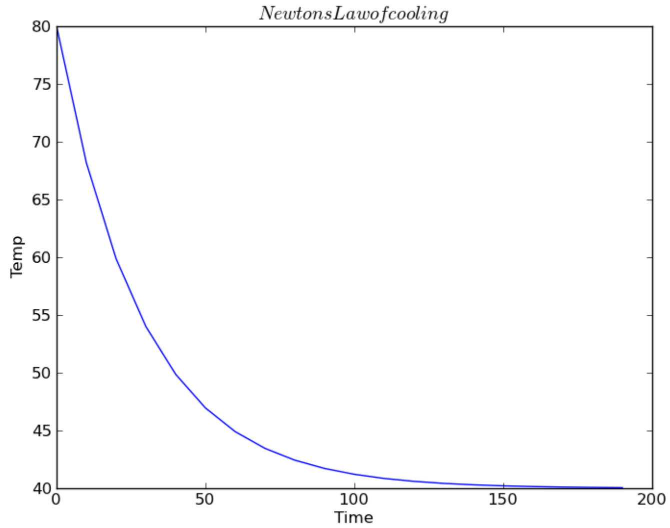
A body originally at $80^\circ c$ cools down to $60^\circ c$ in 20 minutes, the temperature of the air being $40^\circ c$. Then the temperature of the body at ant time t is

$$\theta = 80 + e^{-\frac{log2}{20}t}$$

Programme

$$t = arange(0, 100, 1)$$
$$\theta = 80 + e(-\frac{log2}{20} * t)$$
$$plot(t, \theta)$$
$$xlabel('Time')$$
$$ylabel('Temperature')$$
$$title('Newton's\ Law\ of\ cooling')$$

Graph



$NewtonsLawofcooling$
80
75
70
65
60
55
50
45
40
Temp
0
50
100
150
200
Time

Conclusion:

The simulation of mathematical equations gives insight regarding the behavior of mathematical model. We can trace the value of function at any point. The package Matplotlib is useful in solving trigonometric wave equation and its graphical representation.